Journal of Public Economics 41 (1990) 277–296. North-Holland

TAX REFORM RESULTS WITH DIFFERENT DEMAND SYSTEMS

André DECOSTER and Erik SCHOKKAERT*

Centrum voor Economische Studien, Katholieke Universiteit Leuven, E. van Evenstraat 2B, B-3000 Leuven, Belgium

Received August 1987, revised version received December 1989

We calculate the marginal welfare costs for a twelve-commodity classification of the Belgian indirect tax system. Price elasticities are estimated with Rotterdam, AIDS, CBS and the linear expenditure system. The ranking of the marginal welfare costs is similar when we use unrestricted estimates of the first three systems. After the symmetry condition of the Slutsky matrix is imposed, the differences become more important. However, they diminish with increasing inequality aversion.

1. Introduction

A major obstacle to the practical application of *optimal taxation* theory is the dependence of the results on the specification of the demand system used to estimate the reactions of the consumers. [See, for example, Deaton (1981) and, for an empirical illustration, Ray (1986).] There are many reasons, however, to hypothesize that the problem might be less severe in the context of *marginal tax reform*. This is emphasized by Ahmad and Stern (1984), who argue that for the tax reform problem we need only information about aggregate demand derivatives for the point at which we find ourselves. It is not necessary to have information about individual reactions, nor to use fitted values for configurations away from the starting point. Basically the same argumentation is given by Deaton (1987): he shows the consequences of introducing simplifying assumptions (such as direct or indirect additivity) into the tax reform framework, but he then suggests that second-order flexible functional forms may be sufficiently flexible to yield acceptable estimates for evaluating tax reform proposals. All of this rests more on (reasonable) intuition than on theoretical proof, however. It seems therefore interesting to present some empirical evidence on the subject.

This is the only ambition of this paper. We remain within the framework

*We would not have written this paper without the stimulating presence of A.P. Barten and the easy availability of his computer program, DEMMOD. We would also like to thank Leon Bettendorf for his extremely valuable help with the computations and two anonymous referees for useful comments. This paper contains only a small fraction of the tables that have been produced. More information is available from the authors.

0047-2727/90/$3.50

proposed by Ahmad and Stern (1984, 1987). This approach is summarized in section 2, where we also give an overview of our data. These refer to the Belgian indirect tax system, and we work with twelve aggregate commodities. In section 3 we present the four estimated demand systems: Rotterdam, AIDS, CBS and the linear expenditure system. For the three former systems we have estimated different variants, including complete preference independence for the Rotterdam version. In section 4 we compare the tax reform results with the different systems. Section 5 concludes. As the main objective of this paper is methodological, we will not pay much attention to the policy conclusions of our calculations. These are discussed in more detail in a companion paper [Decoster and Schokkaert (1989)].

2. Tax reform formulae

Because of the limitations of the data, we have concentrated on indirect taxes only. We assume factor income to be fixed and untaxed. Producer prices $p_1,\ldots,p_N$ of the N commodities are also fixed and there are no pure profits. Indirect taxes $t_1,\ldots,t_N$ therefore increase the consumer prices $q_1,\ldots,q_N$:

$$q_i = p_i + t_i, \quad i=1,\ldots,N. \tag{1}$$

There are H households in the economy. Total consumption of commodity i is given by

$$X_i = \sum_h x_i^h, \quad i=1,\ldots,N. \tag{2}$$

where x_i^h is the amount purchased by household h, and is a function of the consumer prices. The government collects a given amount of revenue R through indirect taxes:

$$R = \sum_i t_i X_i, \tag{3}$$

and it cares about social welfare, represented by the function $W[v^1(q),\ldots,v^H(q)]$, where $v^h(q)$, $h=1,\ldots,H$, refers to the indirect utility function of household h.

This is the model proposed by Ahmad and Stern (1984, 1987) where they define MC_i, the marginal cost in terms of social welfare of an extra franc raised via the ith good, as:

$$MC_i = -\frac{\partial W/\partial t_i}{\partial R/\partial t_i}. \tag{4}$$

If $MC_i < MC_j$, social welfare will be increased by lowering the tax rate t_j and by an offsetting increase in the tax t_i, so as to keep global tax revenue constant.

It is immediately obvious that within this simplified model of the economy, (4) can be specified as:

$$MC_i = \frac{\sum_h \beta^h x_i^h}{X_i + \sum_k t_k \left(\frac{\partial X_k}{\partial t_i}\right)}, \tag{5}$$

where $\beta^h = (\partial W/\partial v^h)(\partial v^h/\partial m^h)$, m^h being the lump-sum income of household h. The parameter β^h therefore gives the marginal social valuation of one unit of income accruing to household h.

Multiplying numerator and denominator in (5) by q_i leads to an expression which can be operationalized easily:

$$MC_i = \frac{\sum_h \beta^h (q_i x_i^h)}{q_i X_i + \sum_k \varepsilon_{ki} t_k^* (q_k X_k)}, \tag{6}$$

where ε_{ki} refers to the uncompensated price elasticity and $t_k^* = t_k/q_k$, the tax rate as a fraction of consumer price.

To apply (6) we have to combine information from different sources. We constructed twelve aggregate commodities in such a way that it was possible to match time series information from the Belgian National Accounts and cross-section material from the Belgian consumer expenditure survey 1978/1979. On the basis of the latter, we constructed the consumption data $(q_i x_i^h)$ for ten representative consumers, each corresponding to one decile of the total expenditure distribution in the survey. The indirect tax rates t_i^* were computed from the more detailed study of Paraire-Laguesse et al. (1986). The commodity classification and the tax rates are shown in table 1.

To calculate the welfare weights β^h, we follow Ahmad and Stern and start from the well-known iso-elastic specification for the social welfare function. Under the normalization $\beta^1 = 1$, this leads to

$$\beta^h = \left(\frac{m^h}{m^1}\right)^{-e}. \tag{7}$$

Table 1
Commodity classification and indirect tax rates.

Commodity (abbreviation)	Expenditures average consumer (1)	Value added tax (2)	Excise tax (3)	Indirect tax rate (in %) [(2)+(3)]/(1)
Food (FOOD)	111,531	6,695	191	6.1741
Beverages (BEVE)	16,519	2,874	2,803	34.3665
Tobacco (TOBA)	6,918	392	4,339	68.3868
Clothing (CLOT)	49,145	7,436	–	15.1307
Rent (RENT)	115,303	1,146	–	0.9939
Heating (HEAT)	51,114	6,941	–	13.5794
Durables (DURA)	43,381	7,391	–	15.2767
Housing (HOUS)	12,340	885	–	7.1718
Personal care (PERS)	27,144	2,077	–	7.6518
Transportation (TRANS)	72,001	12,742	10,329	32.0426
Leisure goods (LEIS)	45,648	5,319	–	11.6522
Services (SERV)	9,368	135	–	1.4411

Eq. (7) immediately shows that with an increase in the parameter of inequality aversion e, higher income households get a relatively smaller welfare weight.

For the application of (7) we defined m^h to be total expenditures per equivalent adult.[1] It is well known that the use of such equivalence scales implies a specific cardinalization of the utility function [see, for example, Deaton and Muellbauer (1980a) and Fisher (1987)].

Demand responses appear in (6) through the aggregate price elasticities. To compute the complete matrix of cross-price elasticities we estimated a complete demand system with data from the National Accounts. So doing, we neglected the effect of preference heterogeneity, following, for example, from differences in household size and composition. The available data do not allow us to take into account differences in price elasticities for different groups of consumers. Actually, it is not the aim of this paper to contribute to the solution of the aggregation problem in demand theory. Aggregate estimates have been widely used in the tax reform literature and we simply wanted to investigate the extent to which tax conclusions depend on the specification of the demand system used to estimate these aggregate price elasticities. In this respect, it should also be remembered that the Ahmad/Stern approach evaluates *marginal* tax reforms. Demand reactions do not appear in the social welfare evaluation [the numerator of (6)], but are only important to calculate the marginal effects on tax revenue [the denominator in (6)]. This makes the use of simple aggregate elasticities less worrisome.

[1]The equivalence scales used by the Belgian National Institute of Statistics are those of the League of Nations. These are far from perfect, of course. The welfare weights are not very sensitive to the choice of scale, however. For instance, use of total expenditure per capita leads to a similar pattern of welfare weights.

Moreover, we only need information about demand reactions for the actual situation we want to evaluate. It is not necessary to use the demand system to predict behaviour at other price–income configurations. In our case, we evaluate the elasticities at the average budget shares, taken from the Consumer Expenditure Survey.

3. Demand systems: Specification and estimation

Tax analysts have often used the linear expenditure system (LES) to estimate price elasticities. However, Atkinson (1977) has shown that the use of this system basically determines the outcomes of the optimal tax calculations and Deaton (1987) has pointed to analogous problems for a tax reform analysis. We will show the results for the LES, but only as a benchmark for comparison with other systems. We will devote more attention to the results with three flexible functional forms: Rotterdam, AIDS and CBS.

> Such specifications ensure at least the possibility of a local approximation to whatever the demand functions happen to be, and they guarantee that, at least as far as income and the matrix of price elasticities are concerned, our measurements are indeed measurements and not prior assumptions in disguise [Deaton (1987, p. 101)].

Of course, other flexible functional forms have been proposed in the literature. The most popular one probably is the translog – see Christensen, Jorgenson and Lau (1975). Since all these systems are only approximations it is impossible to determine a priori which one is preferable. Moreover, since we only need estimates of aggregate price responses, direct parameterizations of the demand system such as Rotterdam or CBS are no worse than utility-consistent specifications, such as AIDS or translog [see Deaton (1987)]. With our choice of estimated systems, we do not intend to claim the superiority of any specification. The systems estimated have been widely used, however, and we feel that a comparison of tax reform results with these systems can give a first indication of the sensitivity of policy conclusions with respect to the specification of the demand system.

The first system estimated is the Rotterdam model [see, for example, Theil (1975/1976), Barten and Geyskens (1975)]:

$$\bar{w}_{it}\Delta \ln X_{it} = b_i \sum_j \bar{w}_{jt}\Delta \ln X_{jt} + \sum_j s_{ij}\Delta \ln q_{jt} + v_{it}, \tag{8}$$

where w_{it} is the budget share of commodity i in period t and $\bar{w}_{it} = (w_{i,t} + w_{i,t-1})/2$. The vector b and the matrix S are fixed coefficients to be estimated and interpreted as marginal budget shares and Slutsky coefficients, respectively. The restrictions of demand theory can be formulated immediately in terms of these coefficients:

$\iota' b = 1$ and $\iota' S = 0$	(adding up),
$S\iota = 0$	(homogeneity),
$S = S'$	(symmetry),
S negative semi-definite	(negativity).

We will compare the tax reform results for these different variants. An interesting feature of the Rotterdam model is the possibility of imposing additivity of the utility function (or 'complete preference independence' in the Rotterdam jargon) within the more general model.

A second flexible functional form specification is Deaton and Muellbauer's (1980b) 'almost ideal demand system'. This is specified as follows:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln q_{jt} + \beta_i \ln \{m_t / P_t\} + u_{it}, \tag{9}$$

where $\ln P_t$ is a price index defined as:

$$\ln P_t = \alpha_0 + \sum_k \alpha_k \ln q_{kt} + \tfrac{1}{2} \sum_j \sum_k \gamma_{kj} \ln q_{jt} \ln q_{kt}. \tag{10}$$

This expression makes (9) nonlinear and estimation becomes easier when we approximate it by

$$\ln P_t^* = \sum_k w_{kt} \ln q_{kt}. \tag{11}$$

The use of (11) is an *empirical* approximation. It does not lead to important changes in the results when the prices are collinear [see Deaton and Muellbauer (1980b)]. Using (11) in (9), taking differences and replacing w_{kt} by $\bar{w}_{kt}$, we get:

$$\Delta w_{it} = \beta_i \left(\Delta \ln m_t - \sum_k \bar{w}_{kt} \Delta \ln q_{kt} \right) + \sum_j \gamma_{ij} \Delta \ln q_{jt} + u'_{it}. \tag{12}$$

Since we know that the term in brackets on the right-hand side of (12) is equal to $\sum_k \bar{w}_{kt} \Delta \ln X_{jt}$, we see that the differential form of AIDS leads to the same right-hand side as the Rotterdam model (8).

Of course, the interpretation of the coefficients in (8) and (12) is not the same. It is easy to show that

$$\beta_i = b_i - w_i \tag{13}$$

and

$$G = [\gamma_{ij}] = S + \hat{w} - ww', \tag{14}$$

where $\hat{w}$ is a diagonal matrix with the budget shares on the diagonal. Expression (14) implies that (global) concavity of the cost function cannot be imposed on AIDS. The other restrictions can be written as:

$$\iota'\beta = 0 \quad \text{and} \quad \iota' G = 0 \quad \text{(adding up)},$$

$$G\iota = 0 \qquad \text{(homogeneity)},$$

$$G = G' \qquad \text{(symmetry)},$$

In an attempt to combine the nice features of both models, Keller and Van Driel (1985) proposed the so-called CBS model. This can again be obtained by a change in the dependent variable of (8). Starting from (8), subtracting $\bar{w}_{it} \sum_j \bar{w}_{jt} \Delta \ln X_{jt}$ and using (13), one gets:

$$\bar{w}_{it}\left(\Delta \ln X_{it} - \sum_j \bar{w}_{jt} \Delta \ln X_{jt} \right) = \beta_i \sum_j \bar{w}_{jt} \Delta \ln X_{jt} + \sum_j s_{ij} \Delta \ln q_{jt} + v_{it}. \tag{15}$$

This specification combines the total expenditure coefficients of AIDS with the matrix of price coefficients of the Rotterdam model.

The feature that the Rotterdam model (8), the AIDS specification (12) and the CBS system (15) all have the same right-hand side is exploited in the computer program DEMMOD, constructed by A. P. Barten. This program allows us to estimate all variants of the three systems with the maximum likelihood procedure, described in Barten and Geyskens (1975). To impose

Table 2
Testing the restrictions from demand theory.

Testing homogeneity

Demand system	Laitinen test-statistic	F(11,7) critical value 0.05	0.01	Conclusion
Rotterdam	2.58589	3.6018	6.5382	Homogeneity not rejected
AIDS	2.92910	3.6018	6.5382	Homogeneity not rejected
CBS	2.76468	3.6018	6.5382	Homogeneity not rejected

Testing symmetry

Demand system	2LL[a]	χ^2 0.05	0.01	Conclusion	2LL corrected[b]	Conclusion
Rotterdam	118.737712	65.09	73.57	Rejected	60.805188	Not rejected
AIDS	110.814600	65.09	73.57	Rejected	56.747799	Not rejected
CBS	115.491486	65.09	73.57	Rejected	59.142823	Not rejected

Testing negativity

Demand system	2LL[a]	χ^2 0.05	0.01	Conclusion	2LL corrected[b]	Conclusion
Rotterdam	47.476290	9.49	13.28	Rejected	27.822127	Rejected
CBS	64.307626	9.49	13.28	Rejected	37.685652	Rejected

[a]Likelihood ratio test statistic.
[b]Likelihood ratio test statistic after small-sample correction suggested by Italianer (1985).

symmetry and (especially) negativity, this procedure uses the Choleski decomposition of the Slutsky matrix. We used this program to estimate the three systems for the twelve commodities classification, described in table 1. In all three cases we added an intercept, to be interpreted as a trend shift in demand. The hypothesis that this intercept could be omitted was strongly rejected.

It is perhaps interesting first to have a look at the results for the statistical testing of the different theoretical restrictions. These results are shown in table 2. For this exercise we have to take into account that the usual test statistics (such as the likelihood ratio) are strongly biased towards rejection of the restriction for small samples.[2] For homogeneity we give the exact small sample test proposed by Laitinen (1978): with this exact test, the restriction is rejected for none of the three systems. An exact small sample test for symmetry has not yet been derived. The traditional likelihood ratio tests indicate rejection of the restriction for the three systems. A small-sample

[2]An overview of this problem can be found in Bewley (1986).

correction for these tests has been proposed by Italianer (1985): when applying this correction factor, symmetry passes for the three systems. None of the three symmetric systems satisfies the negativity condition, however. This condition cannot be imposed in AIDS. For Rotterdam and CBS, this inequality restriction can be imposed by requiring the Choleski values to be negative. Table 2 shows that this condition is rejected for both systems, even after the small sample correction.[3] However, violation of the concavity of the cost function is a rather disturbing feature for applied welfare analysis and we will therefore work further with the Rotterdam and CBS estimates under negativity.[4] We return to the problem in the following section.

While the previous conditions directly follow from theory, additivity of the utility function is an *empirical* restriction. It is well known to have immediate consequences for the results on taxation. We therefore estimated an additive system as a standard of comparison for the other systems. This can only be done for the Rotterdam system. When we start from the system under negativity as the maintained hypothesis, additivity is accepted after the small-sample correction. This may seem surprising, but one should remember that negativity already was strongly rejected by our data.

Given the different interpretation of the coefficients, it seems advisable to compare the results for the different systems in elasticity terms. As noted in the previous section, these elasticities were evaluated at the average budget shares, taken from the Consumer Expenditure Survey.[5] In table 3 we show the total expenditure elasticities for ROTT4 (negativity), AIDS3 (symmetry), CBS4 (negativity), ROTT9 (additivity) and the linear expenditure system. Table 4 summarizes all price elasticities for the former three systems. The problems with AIDS immediately show up with the positive own-price coefficients for some noninferior goods.

There are some striking differences between the systems: as could be expected, total expenditure elasticities are similar between CBS and AIDS, while for the price elasticities the resemblance is between CBS and Rotterdam. Of course, the different specifications are not consistent with each other: for example, it is clear from (13) that both b_i and β_i cannot be constant. Since the models are not nested, the choice between them is not an easy matter. The limitations of LES are well known, of course. But the other three

[3]It is not obvious how to determine the number of degrees of freedom for this test because the negativity condition is an inequality restriction. Following Barten and Geyskens (1975), we chose the number of positive Choleski values under symmetry.

[4]One referee suggests that imposing negativity can have the effect of leading very close to homotheticity. However, this result is not found with the specifications that we estimate. It is indeed obvious from table 3 that the estimated expenditure elasticities are not at all equal to unity.

[5]Shares computed with the National Accounts data for the same years 1978/1979 are very similar.

Table 3
Total expenditure elasticities.

	Rotterdam	AIDS	CBS	Complete preference independence (Rotterdam)	LES
FOOD	0.654923	0.597821	0.498945	0.996022	0.534537
BEVE	1.949600	0.957932	1.106894	1.570287	1.561262
TOBA	0.484472	−0.205224	−0.228537	1.521314	0.410965
CLOT	1.530910	1.493856	1.536456	1.887127	0.732513
RENT	0.013764	0.496411	0.484981	0.000231	0.469456
HEAT	1.217049	1.567253	1.483557	0.797202	1.087959
DURA	3.016132	2.766847	2.752187	3.542886	1.495286
HOUS	1.355625	0.533418	0.720986	0.969286	0.820176
PERS	1.387799	0.709662	0.902378	0.253219	2.642756
TRAN	0.679130	0.746790	0.809635	0.659267	1.287659
LEIS	0.682413	0.990877	0.923185	0.600703	0.792828
SERV	6.909038	3.905373	4.412752	4.150863	7.361978

systems all are empirical approximations, and there are no convincing arguments to claim that one of them is superior within the range of observed values for the different variables. Each of the systems has its own supporters among demand specialists and the performance of the specifications seems to vary from case to case. This makes it all the more important to see whether the different estimates for the price elasticities lead to large differences in the estimates of the marginal cost of taxation, as presented in section 2.

4. Tax reform results with the different demand systems

We will proceed in three steps. First, we analyse the results for the case where the inequality aversion parameter e is zero. In that case eq. (6) reduces to:

$$MC_i = \frac{1}{1+\sum_k \varepsilon_{ki} t_k^* \left(\dfrac{q_k X_k}{q_i X_i}\right)}, \tag{16}$$

and the price effects play the dominant role in the explanation of differences between the MC_i's. In this first step, we compare the results for the 'best' variants of the systems, i.e. ROTT4, AIDS3 and CBS4. As standards of comparison we also introduce ROTT9 (Rotterdam under complete preference independence), the linear expenditure system and the 'easy' practical solution where the estimation of demand systems is made unnecessary by the assumption

$$\left.\begin{aligned}\varepsilon_{ii}&=-1\\ \varepsilon_{ik}&=0\end{aligned}\right\},\quad \forall i \neq k. \tag{17}$$

We will call this the 'diagonal' assumption. In a second step we stick to eq. (16), but we will compare the results for the different variants of the systems. We there return to the problem of the previous section about imposing the negativity condition. In the final step we consider what the consequences are of increasing e.

What matters for the tax reform problem is the *ranking* of the different marginal welfare costs. In fact, the numerical values of these costs are dependent on the normalization, chosen for the β^{h}'s. We will therefore summarize our comparisons with Spearman rank correlation coefficients.

4.1. Comparison of results for the case $e=0$

The results for the case $e=0$ are shown in table 5. For some commodities the estimates seem quite different between the systems, striking examples being 'beverages' and, especially, 'services'. The results for the linear expenditure system differ considerably from these for the other systems. Among the flexible forms, the outlier is AIDS, which is not surprising given the *positive* own-price elasticities for these commodities in that system. Remember that for both CBS and Rotterdam, negativity has been imposed. On the other hand, it is easily possible to draw some generally valid policy conclusions from table 5: one could say that the marginal welfare cost for increasing taxes on tobacco and transportation (where Belgium has now already important excise taxes) is high for all sets of estimates.

In table 5 we give, in parentheses, the estimated standard errors of the marginal costs.[6] These are small for the estimates based on the linear expenditure system. Yet they are relatively large with the other systems, at least for some commodities (not surprisingly beverages and services, but also tobacco). This implies that one has to be careful when drawing conclusions

[6]Write eq. (6) as $MC_i=f_i(\varepsilon)$, where ε is the vector of price elasticities. These price elasticities are the only stochastic components in eq. (6). The covariance matrix V of the marginal costs then has been computed through the first order-approximation $[v_{ij}]=[\partial f_i/\partial\varepsilon]'V_\varepsilon[\partial f_j/\partial\varepsilon]$, where V_ε is the covariance matrix of the price elasticities. Treating the budget shares as nonstochastic, this latter matrix can be calculated directly from the covariance matrix of the estimated coefficients for Rotterdam, AIDS and CBS. The DEMMOD program gives an estimate of this covariance matrix of the coefficients for the homogeneity variant. This is an asymptotically consistent estimator for the covariance matrices under symmetry and negativity. For the linear expenditure system, the elasticities are a nonlinear function of the estimated coefficients. Their covariance matrix then also has been computed through a first order-approximation.

Table 4
Price elasticities.

	FOOD	BEVE	TOBA	CLOT	RENT	HEAT	DURA	HOUS	PERS	TRAN	LEIS	SERV
						ROTT4						
FOOD	−0.67	−0.02	0.01	0.02	−0.14	−0.05	0.03	0.05	0.05	−0.06	−0.00	0.12
BEVE	−0.38	−0.14	−0.02	0.25	−0.33	−0.25	−0.27	−0.09	−0.05	−0.40	−0.25	0.01
TOBA	0.20	−0.01	−0.94	−0.27	−0.24	−0.02	0.67	0.23	0.62	0.12	−0.69	−0.15
CLOT	−0.14	0.09	−0.05	−0.80	−0.32	0.05	0.10	−0.01	−0.19	−0.06	−0.14	−0.06
RENT	−0.02	0.01	−0.01	−0.01	−0.03	0.02	−0.03	0.02	0.01	0.07	−0.02	−0.02
HEAT	−0.25	−0.09	−0.01	0.10	−0.17	−0.36	−0.05	−0.02	−0.06	−0.28	−0.02	−0.00
DURA	−0.39	−0.12	0.07	−0.02	−0.65	−0.16	−0.78	0.03	−0.25	−0.16	−0.43	−0.15
HOUS	0.22	−0.08	0.09	−0.02	−0.09	−0.05	0.23	−0.96	−0.13	−0.32	−0.11	−0.14
PERS	0.04	−0.01	0.12	−0.29	−0.25	−0.09	−0.23	−0.06	−0.28	−0.21	−0.08	−0.07
TRAN	−0.09	−0.05	0.01	0.03	−0.01	−0.11	0.08	−0.05	−0.06	−0.56	−0.02	0.16
LEIS	−0.01	−0.03	−0.07	−0.03	−0.17	0.02	−0.12	−0.01	0.00	−0.02	−0.19	−0.06
SERV	1.00	−0.12	−0.24	−1.00	−1.80	−0.38	−1.52	−0.50	−0.67	1.16	−1.32	−1.51
						AIDS3						
FOOD	−0.46	−0.05	−0.01	−0.01	−0.04	0.06	−0.04	0.01	−0.04	−0.12	0.03	0.06
BEVE	−0.41	0.19	0.07	0.25	−0.56	−0.20	−0.36	−0.04	−0.12	0.06	0.22	−0.05
TOBA	0.03	0.20	−0.20	−0.44	−0.15	−0.00	0.67	0.51	0.75	0.33	−1.22	−0.28
CLOT	−0.20	0.06	−0.08	−0.65	−0.31	0.09	0.11	−0.05	−0.25	−0.04	−0.11	−0.10
RENT	−0.02	−0.07	−0.02	−0.05	−0.40	0.00	−0.07	0.05	0.02	0.14	−0.03	−0.06
HEAT	−0.00	−0.10	−0.02	0.11	−0.21	−0.41	0.03	−0.09	−0.29	−0.44	0.01	−0.17
DURA	−0.53	−0.17	0.06	0.01	−0.61	−0.05	0.13	0.02	−0.26	−0.17	−0.72	−0.47
HOUS	0.10	−0.03	0.22	−0.08	0.37	−0.16	0.25	−0.02	−0.13	−0.14	−0.52	−0.41
PERS	−0.18	−0.05	0.15	−0.33	0.03	−0.28	−0.21	−0.06	0.20	−0.13	−0.01	0.18
TRAN	−0.22	0.03	0.02	0.06	0.17	−0.18	0.06	−0.03	−0.06	−0.69	−0.07	0.18
LEIS	−0.03	0.05	−0.14	−0.04	−0.15	0.04	−0.37	−0.13	−0.02	−0.10	−0.15	0.05
SERV	0.55	−0.22	−0.36	−1.08	−1.74	−1.23	−3.79	−1.12	0.77	1.73	0.17	2.40

	CBS4											
FOOD	−0.28	−0.08	0.03	0.09	−0.14	−0.04	−0.11	0.04	0.01	0.01	−0.03	0.01
BEVE	−0.70	−0.21	0.12	0.20	−0.33	−0.09	−0.27	0.09	0.01	0.06	−0.01	0.02
TOBA	0.58	0.31	−0.92	−0.05	−0.24	0.07	0.09	0.52	0.37	−0.06	−0.54	0.04
CLOT	−0.00	0.05	−0.03	−0.79	−0.32	0.04	−0.00	0.01	−0.23	−0.06	−0.14	−0.05
RENT	−0.14	−0.03	−0.02	−0.06	−0.14	−0.00	−0.07	0.04	−0.01	0.01	−0.06	0.00
HEAT	−0.32	−0.05	−0.01	0.06	−0.21	−0.33	−0.02	−0.00	−0.07	−0.44	−0.08	−0.01
DURA	−0.72	−0.14	−0.02	−0.11	−0.61	−0.10	−0.47	−0.02	−0.02	−0.04	−0.43	−0.07
HOUS	0.24	0.11	0.22	0.12	0.25	0.04	0.11	−0.85	−0.26	−0.48	−0.13	−0.09
PERS	−0.05	0.01	0.06	−0.31	−0.12	−0.04	0.13	−0.13	−0.23	−0.29	0.03	0.03
TRAN	−0.05	0.02	−0.01	0.02	−0.05	−0.19	0.13	−0.10	−0.12	−0.50	−0.02	0.07
LEIS	−0.13	0.00	−0.07	−0.06	−0.19	−0.01	−0.15	−0.03	0.01	−0.04	−0.22	−0.03
SERV	−0.60	−0.03	−0.01	−0.69	−0.75	−0.25	−0.70	−0.31	−0.04	0.37	−0.74	−0.65

Table 5
MC_i's and rankings for $e=0$.

Commodity	DIAGONAL		ROTT4		AIDS3		CBS4		ROTT9		LES	
FOOD	1.0658	10	1.1261 (0.0439)	8	1.1648 (0.0501)	5	1.1183 (0.0456)	8	1.1323 (0.0444)	11	1.1544 (0.0022)	8
BEVE	1.5236	2	1.2429 (0.4494)	3	0.9438 (0.2764)	12	1.0310 (0.3262)	11	1.3759 (0.5507)	2	1.5235 (0.0636)	1
TOBA	3.1632	1	2.7963 (3.2585)	1	1.1548 (0.5929)	6	2.7411 (3.3028)	1	1.9117 (1.5230)	1	1.4901 (0.0318)	2
CLOT	1.1783	5	1.1336 (0.0539)	7	1.1106 (0.0552)	8	1.1297 (0.0565)	7	1.1790 (0.0584)	6	1.1499 (0.0075)	9
RENT	1.0085	12	1.1476 (0.0443)	6	1.0893 (0.0425)	9	1.1500 (0.0469)	5	1.1812 (0.0469)	4	1.0603 (0.0003)	12
HEAT	1.1571	6	1.2301 (0.0705)	4	1.2316 (0.0754)	4	1.2294 (0.0743)	3	1.1734 (0.0641)	7	1.1304 (0.0096)	10
DURA	1.1803	4	1.0584 (0.1280)	11	0.9868 (0.1187)	11	1.0759 (0.1395)	9	1.1799 (0.1591)	5	1.1880 (0.0165)	7
HOUS	1.0773	9	1.1084 (0.4671)	9	1.0289 (0.4293)	10	1.0494 (0.4416)	10	1.1385 (0.4928)	10	1.3246 (0.0053)	3
PERS	1.0829	8	1.0930 (0.1580)	10	1.1396 (0.1832)	7	1.1434 (0.1824)	6	1.1704 (0.1812)	8	1.2155 (0.0152)	6
TRAN	1.4715	3	1.3574 (0.1187)	2	1.3677 (0.1285)	1	1.2774 (0.1108)	2	1.2462 (0.1000)	3	1.2610 (0.0335)	5
LEIS	1.1319	7	1.2071 (0.1292)	5	1.2589 (0.1498)	3	1.1745 (0.1290)	4	1.1689 (0.1211)	9	1.1158 (0.0059)	11
SERV	1.0146	11	0.8096 (0.3818)	12	1.3633 (1.1549)	2	0.8998 (0.4974)	12	0.9277 (0.5014)	12	1.3149 (1.1774)	4

Ranking of commodities

	DIAGONAL	ROTT4	AIDS3	CBS4	ROTT9	LES
Highest	TOBA	TOBA	TRAN	TOBA	TOBA	BEVE
marginal	BEVE	TRAN	SERV	TRAN	BEVE	TOBA
welfare	TRAN	BEVE	LEIS	HEAT	TRAN	HOUS
cost	DURA	HEAT	HEAT	LEIS	RENT	SERV
	CLOT	LEIS	FOOD	RENT	DURA	TRAN
	HEAT	RENT	TOBA	PERS	CLOT	PERS
	LEIS	CLOT	PERS	CLOT	HEAT	DURA
	PERS	FOOD	CLOT	FOOD	PERS	FOOD
Lowest	HOUS	HOUS	RENT	DURA	LEIS	CLOT
marginal	FOOD	PERS	HOUS	HOUS	HOUS	HEAT
welfare	SERV	DURA	DURA	BEVE	FOOD	LEIS
cost	RENT	SERV	BEVE	SERV	SERV	RENT

Table 6
Rank correlation between demand models ($e=0$).

	DIAGONAL	ROTT4	AIDS3	CBS4	ROTT9	LES
DIAGONAL	1.0000					
ROTT4	0.6224	1.0000				
AIDS3	−0.1329	0.1608	1.0000			
CBS4	0.3427	0.6923	0.4406	1.0000		
ROTT9	0.7413	0.7133	−0.3147	0.4615	1.0000	
LES	0.4545	0.0979	−0.1958	−0.3217	0.2238	1.0000

about welfare-improving directions of tax reform. For CBS4, for example, there is a probability of about 0.30 that one commits an error by stating that the marginal cost of tobacco is larger than the one of services (one-sided t-test). However, as noted by Ahmad and Stern (1987, p. 301), if the estimated marginal cost of taxing commodity i is larger than the estimated marginal cost of taxing commodity j, then the expected change in welfare from a switch into j is positive; and if the distribution is symmetric, the probability of an increase in welfare is above 50 percent. It therefore still makes sense to look at the rankings of the marginal welfare costs.

The information for the different systems is summarized in table 6, where we give rank correlation coefficients. The linear expenditure and AIDS-systems obviously are outliers. The affinity of Rotterdam and CBS is clear. It is striking also that the 'simple' solutions (DIAGONAL and ROTT9) are not completely divergent from the more complex systems. It is probably fair to conclude that the differences between the results are not as large as could be expected when considering table 4. There is some truth in Ahmad and Stern's (1984, p. 293) suggestion that this might be explained by the fact that all the demand effects are summed to compute (16). On the other hand, our results also suggest that the practitioner who would use information from one system only (e.g. AIDS under symmetry) cannot be confident that his results are not influenced heavily by this (more or less arbitrary) choice of system. We will now see whether this result is due to the fact that we compare AIDS under symmetry with the other systems under negativity.

4.2. Results for the different variants ($e=0$)

Table 7 shows the rankings of welfare costs for the different variants of the flexible functional form systems and the corresponding rank correlation coefficients. The suffixes 1 to 4 refer to the variants 'free', 'under homogeneity', 'under homogeneity and symmetry', and 'under homogeneity, symmetry and negativity', respectively. We see that the divergent behaviour of AIDS is also present when we compare AIDS3 with CBS3 and ROTT3. In fact, table 7 suggests some further interesting conclusions.

Table 7

Rankings for all variants ($e=0$).

ROTT1	ROTT2	ROTT3	ROTT4	AIDS1	AIDS2	AIDS3	CBS1	CBS2	CBS3	CBS4
TOBA	TOBA	TOBA	TOBA	TOBA	TOBA	TRAN	TOBA	TOBA	TOBA	TOBA
BEVE	BEVE	TRAN	TRAN	BEVE	BEVE	SERV	BEVE	BEVE	LEIS	TRAN
RENT	RENT	LEIS	BEVE	SERV	SERV	LEIS	RENT	HEAT	TRAN	HEAT
HEAT	HEAT	HEAT	HEAT	TRAN	TRAN	HEAT	HEAT	RENT	HEAT	LEIS
TRAN	TRAN	RENT	LEIS	HEAT	HEAT	FOOD	TRAN	TRAN	RENT	RENT
CLOT	CLOT	BEVE	RENT	CLOT	CLOT	TOBA	CLOT	CLOT	BEVE	PERS
DURA	DURA	FOOD	CLOT	RENT	RENT	PERS	DURA	FOOD	FOOD	CLOT
FOOD	FOOD	CLOT	FOOD	FOOD	FOOD	CLOT	FOOD	DURA	CLOT	FOOD
LEIS	LEIS	PERS	HOUS	DURA	DURA	RENT	LEIS	LEIS	PERS	DURA
PERS	PERS	HOUS	PERS	LEIS	LEIS	HOUS	PERS	PERS	HOUS	HOUS
SERV	SERV	DURA	DURA	PERS	PERS	DURA	SERV	SERV	DURA	BEVE
HOUS	HOUS	SERV	SERV	HOUS	HOUS	BEVE	HOUS	HOUS	SERV	SERV

Rank correlation coefficients for demand systems

	ROTT1	ROTT2	ROTT3	ROTT4	AIDS1	AIDS2	AIDS3	CBS1	CBS2	CBS3	CBS4
ROTT1	1.0000										
ROTT2	1.0000	1.0000									
ROTT3	0.6783	0.6783	1.0000								
ROTT4	0.7832	0.7832	0.9371	1.0000							
AIDS1	0.6923	0.6923	0.3986	0.5524	1.0000						
AIDS2	0.6923	0.6923	0.3986	0.5524	1.0000	1.0000					
AIDS3	−0.1748	−0.1748	0.3497	0.1608	0.2098	0.2098	1.0000				
CBS1	1.0000	1.0000	0.6783	0.7832	0.6923	0.6923	−0.1748	1.0000			
CBS2	0.9860	0.9860	0.7133	0.8182	0.7133	0.7133	−0.0979	0.9860	1.0000		
CBS3	0.6503	0.6503	0.9930	0.9161	0.3566	0.3566	0.3357	0.6503	0.6853	1.0000	
CBS4	0.4895	0.4895	0.8531	0.6923	0.1608	0.1608	0.4406	0.4895	0.5105	0.8392	1.0000

Free estimates of the different systems lead to very similar rankings of the welfare costs. Imposing homogeneity does not change this picture drastically. In fact, these two variants could be seen as exercises of curve fitting, while homogeneity only imposes one restriction per equation. Symmetry, however, leads to pronounced changes in the rankings, even for the same system: compare ROTT2 and ROTT3, AIDS2 and AIDS3, and CBS2 and CBS3. At the same time, differences between the different systems also grow larger. There still is a close connection between CBS3 and ROTT3, however.

This may be the place to return to a question, raised earlier: Should one impose the theoretical restrictions? In our case, at least three possible positions can be taken. A first one could be called the *theoretical* position. The theoretician will argue that welfare-economic applications run into deep trouble when estimated demand parameters are not consistent with a concave cost function. He will point out that there are so many data limitations and remaining theoretical problems[7] that one should not attach too much attention to the statistical rejection of the negativity condition. In our case, the theoretician has to face the problem that to a certain extent results will be dependent on the choice of specification. Either he will have to compare different systems in order to draw conclusions which hold for all (as is possible with our results), or he will have to argue on theoretical grounds that one system is superior. A second position could be called the *statistical* one. Someone holding this position will not want to impose theoretical restrictions if they are rejected by the data. In our case, he will stick to the symmetry versions,[8] and he will have to face the same choice problem as the theoretician. A third position is the *pragmatic* one. The pragmatist will not like to be faced with a choice between different systems. Like the theoretician he will point to the many problems (both theoretical and practical) arising when estimating a fully-fledged complete demand system. He will argue that we do not need accurate estimates of all cross-price effects and that to apply (16) only the sum of these effects is necessary. Why bother about the precise estimation of cross-price effects if we can only estimate them conditional upon the 'arbitrary' choice of system? After all, one can have doubts about the validity of the theory. The pragmatist probably will be happy with the finding that a free curve-fitting exercise (even after imposing the homogeneity condition) yields results which are rather insensitive to the exact specification used and could prefer these nonsymmetric estimates.

While endorsing the theoretical position ourselves, we feel that more

[7]For an overview see Deaton and Muellbauer (1980a, pp. 78–82).

[8]Note that Wibaut (1987) in his empirical application for Belgium uses estimates from a Rotterdam system with symmetry imposed, but with a positive diagonal element in the Slutsky matrix. He does not explain why he uses that variant.

Table 8
Rank correlations between demand systems.

Correlation between	Value of e						
	0.0	0.1	0.5	1.0	2.0	5.0	10.0
CBS4–ZERO	0.1888	0.1329	0.1678	0.2937	0.4545	0.6593	0.8531
CBS4–DIAGONAL	0.3427	0.3427	0.3776	0.4895	0.5035	0.6154	0.7692
CBS4–ROTT4	0.6923	0.6923	0.6923	0.7823	0.8392	0.8462	0.9301
CBS4–AIDS3	0.4406	0.4406	0.3916	0.3916	0.3846	0.4545	0.6224
CBS4–ROTT9	0.4615	0.4615	0.5804	0.6084	0.7413	0.8042	0.9091
CBS4–LES	−0.3217	−0.3217	−0.2937	−0.2448	−0.0769	0.3427	0.5944
ROTT4–AIDS3	0.1608	0.1608	0.1119	0.2308	0.2937	0.3427	0.4825

Table 9
Rankings for $e=2$.

	ZERO	DIAGONAL	ROTT4	AIDS3	CBS4	ROTT9	LES
1	HEAT	TOBA	TOBA	SERV	TOBA	TOBA	BEVE
2	FOOD	BEVE	HEAT	TRAN	HEAT	BEVE	TOBA
3	TOBA	TRAN	TRAN	HEAT	TRAN	HEAT	SERV
4	BEVE	HEAT	BEVE	LEIS	FOOD	PERS	HOUS
5	PERS	CLOT	LEIS	FOOD	PERS	RENT	PERS
6	SERV	DURA	FOOD	TOBA	RENT	TRAN	FOOD
7	RENT	FOOD	RENT	PERS	LEIS	FOOD	TRAN
8	LEIS	LEIS	PERS	RENT	CLOT	CLOT	HEAT
9	CLOT	PERS	CLOT	CLOT	BEVE	LEIS	DURA
10	HOUS	HOUS	HOUS	HOUS	DURA	DURA	CLOT
11	DURA	SERV	DURA	BEVE	HOUS	HOUS	LEIS
12	TRAN	RENT	SERV	DURA	SERV	SERV	RENT

experience with demand systems will be needed before one can give a fully convincing argumentation for any of these positions.

4.3. Increasing the inequality aversion

Table 8 shows the effects of increasing the inequality aversion parameter e. In this table we use CBS4 as the main standard of comparison and add a variant 'ZERO' which simply assumes that $\varepsilon_{ki}=0$, $\forall k$, i, and hence neglects all changes in demand. Table 9 contains the rankings for $e=2$, which seems to be a 'reasonable' value [see Stern (1977)].

For $e=2$, there still are important differences between the results with the different systems, while as before it is possible at the same time to draw some stable policy conclusions. Table 8 clearly shows that the differences become smaller when e increases. This result is opposite to the one found by Ray (1986) in his computation of 'optimal' taxes. The explanation is obvious, however. When e increases, considerations of distributive justice get a relatively larger weight in (5) and (6). In fact, these considerations become irrelevant when $e=0$ [see (16)]. In the framework of tax reform (in contrast to optimal taxation), justice considerations are captured by actual *observations* of consumption patterns [the numerator in (6)] and, hence, completely independent of the elasticity estimates. For an analysis of tax reform, increasing e therefore amounts to increasing the relative weight attached to actual consumption. A striking illustration of this phenomenon is the high correlation between the results for CBS4 and the results for DIAGONAL and (even!) ZERO.[9]

5. Conclusion

A comparison of the results with the different systems shows that it is possible to draw relevant policy conclusions that hold for all of them. This similarity occurs despite the differences in the matrices of estimated expenditure and price elasticities. From that point of view, it is possible to corroborate the suggestion that the sensitivity to the specification of the demand system is less severe for a tax reform exercise than it is for the calculation of optimal tax rates. This does not mean, however, that the exclusive use of one system always leads to reliable results. Some differences remain large and a sensitivity analysis seems to be advisable. Increasing the inequality aversion implies that a relatively larger weight is given to observed consumption patterns. The choice of demand system therefore becomes less crucial.

It is remarkable how strongly the rankings of the marginal welfare costs are affected by imposing the restriction of symmetry of the Slutsky matrix. Without this integrability condition, the different systems yield very similar results. This could be interpreted by a pragmatist as implying that the estimates of the cross-price effects are heavily influenced by the mathematical straitjacket of the chosen specification, and that the symmetrical estimate of the Slutsky matrix is not really informative about reality. Then why not use the estimates under homogeneity? Surely, more work about specification and estimation of complete demand systems will be necessary if one wants to formulate a completely convincing answer to this pragmatical critique.

[9]Increasing e, i.e. increasing the relative weight attached to actual consumption (a nonstochastic variable), also leads to a considerable decrease in the estimated standard errors of the marginal costs.

References

Ahmad, E. and N. Stern, 1984, The theory of reform and Indian indirect taxes, Journal of Public Economics 25, 259–298.

Ahmad, E. and N. Stern, 1987, Alternative sources of government revenue, in: D. Newbery and N. Stern, eds., The theory of taxation for developing countries (Oxford University Press) 281–332.

Atkinson, A.B., 1977, Optimal taxation and the direct versus indirect tax controversy, Canadian Journal of Economics 10, 590–606.

Barten, A.P. and E. Geyskens, 1975, The negativity condition in consumer demand, European Economic Review 6, 227–260.

Bewley, R., 1986, Allocation models: Specification, estimation and applications (Ballinger, Cambridge).

Christensen, L.R., D.W. Jorgenson and L.J. Lau, 1975, Transcendental logarithmic utility function, American Economic Review 65, 367–383.

Deaton, A., 1981, Optimal taxes and the structure of preferences, Econometrica 49, 1245–1260.

Deaton, A., 1987, Econometric issues for tax design in developing countries, in: D. Newbery and N. Stern, eds., The theory of taxation for developing countries (Oxford University Press) 92–113.

Deaton, A. and J. Muellbauer, 1980a, Economics and consumer behavior (Cambridge University Press).

Deaton, A. and J. Muellbauer, 1980b, An almost ideal demand system, American Economic Review 70, 312–326.

Decoster, A. and E. Schokkaert, 1989, Equity and efficiency of a reform of Belgian indirect taxes, Recherches Economiques de Louvain 55, 155–176.

Fisher, F., 1987, Household equivalence scales and interpersonal comparisons, Review of Economic Studies 54, 519–524.

Italianer, A., 1985, A small-sample correction for the likelihood ratio test, Economics Letters 19, 315–317.

Keller, W.J. and J. Van Driel, 1985, Differential consumer demand systems, European Economic Review 27, 375–390.

Laitinen, K., 1978, Why is demand homogeneity so often rejected?, Economics Letters 1, 187–191.

Paraire-Laguesse, Y. et al., 1986, De weerslag van de indirecte belastingen op de verdeling van de gezinsinkomens in België, Documentatieblad van het Ministerie van Financiën van België 7–8, 9–32.

Ray, R., 1986, Sensitivity of 'optimal' commodity tax rates to alternative demand functional forms, Journal of Public Economics 31, 253–268.

Stern, N., 1977, The marginal valuation of income, in: M. Artis and A. Nobay, eds., Studies in modern economic analysis (Blackwell, Oxford) 209–254.

Theil, H., 1975/1976, Theory and measurement of consumer demand I and II (North-Holland, Amsterdam).

Wibaut, S., 1987, A model of tax reform for Belgium, Journal of Public Economics 32, 53–77.