FEASIBILITY STUDY: HOW TO INTEGRATE LABOUR SUPPLY IN THE BELGIAN TAX BENEFIT MODEL MIMOSIS?

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REPORT 3: FEASIBILITY STUDY - HOW TO INTEGRATE LABOUR SUPPLY IN MIMOSIS?

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FEASIBILITY STUDY: HOW TO INTEGRATE LABOUR SUPPLY IN THE BELGIAN TAX BENEFIT MODEL MIMOSIS?

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Abstract: In this report we outline the steps to be taken after the estimation of the labour supply model (as described in Report 2 of this project) in order to integrate labour supply responses in the MIMOSIS model. We explain the difference between either calibrating the model in the baseline, or directly using the estimated probabilities to obtain expected labour supply, income and other variables. We also sketch briefly the sequence of logical steps to integrate labour supply responses in a policy simulation using pseudo-code.

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1 INTRODUCTION

In this report we outline the steps taken after the estimation of the labour supply model in order to integrate labour supply responses in reform analyses. After the description of the steps taken, we will tentatively list some considerations for the integration of the model in MIMOSIS.

In our view it is possible to integrate labour supply responses in a normal run of MIMOSIS without too much technical difficulties. Of course, this does not liberate the user of the model to first "manually" find a reform that is revenue neutral in a standard simulation, i.e. without behavioural reactions (and if revenue neutrality is required). Integrating labour supply reactions in this reform scenario is then rather straightforward, though computationally more involved than a standard analysis. In what follows we will outline the procedures implemented to evaluate the effects policies have on labour market participation and hence on the cost recovery effect or the degree of self-financing of the reform.

In the remainder it is assumed that the utility function has been estimated and that the coefficients are known (see Decoster, De Swerdt and van Camp 2010 for a description of this methodology with the data underlying MIMOSIS).

2 STEPS AFTER ESTIMATION OF LABOUR SUPPLY MODEL

After the estimation of the parameters of the utility function there are basically two options: calibration of the baseline estimation or no calibration. A model can be calibrated by adding a random error term to the deterministic part of the stochastic utility function such that the choice based on utility corresponds to the choice actually observed, i.e. if a single male works 40 hours a random error term is added such that the utility level corresponding to the observed hours worked is the highest among all hour options. This calibration is not a necessary step and it can be left to the user to switch it on. It need not be an integral part of a simulation. As we will explain in the next section the drawing of the error terms should be done only once after which they are stored in an input file that can then be read in by the model if the user should request the use of a calibrated model. But as said, it is perfectly possible to run a simulation without using calibration and turning calibration off will speed up the calculations. In the next paragraph we explain how calibration works.

2.1 CALIBRATION

The term "calibration" means that, after the estimation, the model is "fine-tuned" such that the prediction of the model corresponds to the actual labour supply observed. This is done by drawing random terms from the assumed distribution of random terms which in essence makes up the stochastic nature of the model. Indeed, the utility level V_{ij} (where i stands for individual i and j represents the choice of hours, i.e. 0, 20 or 40), on which the individual is assumed to base his choice, is defined to consist of both a deterministic part $U(.)$ and a stochastic part, represented by a realisation of an error term ε_{ij}:

$$V_{ij} = U\left(Lf_{ij}, Lm_{ij}, C_{ij}, Z_i\right) + \varepsilon_{ij}. \tag{1}$$

The maximum likelihood estimation results in a set of coefficients for the deterministic part, $U(\ )$. In the model used in Decoster, De Swerdt and van Camp (2010) we chose a quadratic specification for the deterministic part as in (2):

$$\begin{aligned} U_{ij} &= \alpha_c C_{ij} + \alpha_{cc} C_{ij}^2 + \alpha_{lf} Lf_{ij} + \alpha_{llf} Lf_{ij}^2 + \alpha_{lm} Lm_{ij} + \alpha_{llm} Lm_{ij}^2 \\ &\quad + \alpha_{clf} C_{ij} Lf_{ij} + \alpha_{clm} C_{ij} Lm_{ij} + \alpha_{lmlf} Lf_{ij} Lm_{ij} - \boldsymbol{\beta}'_f \mathbf{d}_f - \boldsymbol{\beta}'_m \mathbf{d}_m. \end{aligned} \tag{2}$$

However, using the estimated coefficients and plugging in the values of disposable income and leisure that correspond to each possible choice of hours worked in the deterministic part of the utility function, does not necessarily result in a maximum utility level among all hour options that corresponds with the observed level of labour supply (in other words: the 'fit' is not perfect). That is where calibration enters the scene.

In order for the model to be calibrated an error term, ε_{ij}, is added to the deterministic part of the utility function such that the predicted choice by the model is the same as the choice actually observed. If someone is working 20 hours then the model should be calibrated such that the utility level V_{ij} corresponding to this discrete point is the highest of all the possible choices for this individual.

The random error terms are drawn from the assumed theoretical distribution of the stochastic part of the utility function. In our case this is a type I extreme value distribution. The procedure involves a loop and in each run of the loop an error term is drawn from the theoretical distribution and added to the deterministic part of the utility function corresponding to each possible level of hours worked. We then look at the maximum level of utility and if the number of hours worked for this utility level corresponds to the actual (discretized) number of hours worked the error term is retained, otherwise a new error term is drawn until the number of hours supplied at the maximum level of utility corresponds with observed labour supply. There is thus no predefined end to this loop and it continues until an error term is drawn for each individual in the sample that ensures a predicted value equal to the observed one. This procedure is repeated until we have for each individual a number x of error terms (e.g. $x = 100$), each of which makes the predicted choice equal to the observed one. With calibration, the baseline situation in fact produces a perfect fit of the model.

2.2 REFORM SITUATION

2.2.1 evaluation with calibration

In a reform situation the deterministic part of the utility function is recalculated, given the estimated parameter values. For each possible level of hours worked the disposable income as calculated by MIMOSIS is plugged into the deterministic part of the utility function as is the corresponding amount of leisure time (leisure time in our model is equal to 80 minus hours worked per week). This gives us the utility level corresponding to the deterministic part.

Subsequently a loop is initiated that runs x times, i.e. the number of distinct error terms drawn in the calibration step (100 in our example). In each run of the loop the corresponding error term of the calibration step is added to the deterministic part of the utility function for each potential number of hours worked if the reform scenario. The choice of hours worked that generates the highest (stochastic) utility level is predicted by the model as the optimal choice for this individual.

In short, the first run of the loop adds the first error term that was drawn and retained in the calibration step to U_{i0}^r, U_{i20}^r, and U_{i40}^r if individual i faces a choice set comprised of three distinct levels of hours worked. The superscript r indicates the utility level in the reform scenario. We then determine the maximum as:

$$\max\left(V_{i0}^r, V_{i20}^r, V_{i40}^r\right) = \max\left(U_{i0}^r + \tilde{\varepsilon}_{i1}, U_{i20}^r + \tilde{\varepsilon}_{i1}, U_{i40}^r + \tilde{\varepsilon}_{i1}\right), \quad (3)$$

where $\tilde{\varepsilon}_{i1}$ is the first error term of the calibration step that was retained and that resulted in a maximum (random) utility level V_{ij} such that the number of hours worked, j, corresponds to the discretized number of hours worked observed in the data. Let's assume that we predict the utility level that corresponds with 40 hours per week to be maximal. We then retain 40 hours as the predicted choice and add it to the number of times 40 hours has been predicted as being the optimal choice according to utility maximizing behaviour. We perform this step as many times as we have error terms drawn in the calibration step.

The result then is a number of predictions (100 in our example). The probability of a certain number of hours worked is equal to the number of times this level (or combination of levels in the case of couples) is chosen as optimal divided by 100 (or the number of error terms drawn) and expressed as a percentage. For example, if we predict 40 hours as the optimal level 40 times, 20 hours 50 times and 0 hours 10 times, the probabilities of respectively working full time, half time and not working are then 40%, 50% and 10%. And these probabilities will then be used to determine aggregate expected levels of labour supply and to evaluate the impact of the reform on labour supply and to assess the reform in terms of its self-financing ability or cost recovery effect.

2.2.2 evaluation without calibration

As stated above, using calibration is according to the preferences of the user. A reform can be evaluated perfectly without using calibration. The calibration step, i.e. the determination of error terms, is or can be quite time consuming but as pointed out before is something that can be, and should be, done only once outside of any simulation. It can be done after the estimation of the labour supply model and the resulting error terms stored in an external file that can serve as input for future simulations.

Without calibration the probabilities corresponding to each possible choice in the discrete choice set can be estimated using the theoretical expression in (4) that can be derived when it is assumed that the error terms are identically and independently distributed according to a type I extreme value distribution (McFadden, 1974):

$$P_{ik} = \Pr\left(V_{ik} \geq V_{ij}, \forall j = 0,\dots,J\right) = \frac{\exp U\left(Lf_{ik}, Lm_{ik}, C_{ik}, Z_i\right)}{\sum_{j=0}^{J} \exp U\left(Lf_{ij}, Lm_{ij}, C_{ij}, Z_i\right)}. \qquad (4)$$

Given the parameter estimates and functional form of (the deterministic part of) the utility function, the corresponding probabilities can then be easily calculated using this expression, both in the baseline and in the reform situation. The resulting probabilities can subsequently be used to assess the aggregate labour supply effects of any reform and the effects on government revenue by multiplying the numbers (taxes, social insurance contributions, etc) in each discrete point with the corresponding probability that that point is chosen. Remark that it is advisable to compare the aggregate numbers to a baseline that is calculated in the same way, i.e. by using probabilities and not by using the actual observed numbers.

3 INTEGRATION IN MIMOSIS

From the above it is clear that it should not be too difficult to integrate this in MIMOSIS once the parameter estimates of the utility function and optionally the random error terms are known.

What is fixed once the model has been estimated?

– the parameter estimates of the utility function

– the error terms

What is needed for a reform?

– the parameters of the utility function

– the simulated incomes in the reform situation

– the error terms (optional)

As explained above what then needs to be done in a reform is determine the probabilities given the parameter estimates (and optionally error terms) and the simulation results (incomes) and calculate aggregate expected values for the variables of interest, e.g. government revenue.

What if we want revenue neutrality? Can this be automated? It probably can, but it would be time consuming, even if a certain deviation range is allowed for. But revenue neutrality is something that should hold in the static situation, i.e. the situation without labour supply responses, and it is something that is not automated in the standard version of MIMOSIS either. Revenue neutrality has to be guesstimated by the user by trial and error or some other informed methodology. Once revenue neutrality is obtained in a standard simulation, labour supply responses can be integrated to assess the "pure" cost recovery effects of the reform.

If the functional specification of the utility function is agreed upon, the parameter estimates together with error terms can be stored in an input file (or two input files).[1] One can even have several input files corresponding to different definitions of discrete choice sets if some flexibility in the choice of discretization is wanted.

Basically, once the model has been estimated the following steps are mere calculations that can straightforwardly be programmed in a separate module. The calibration option would be computationally more involved as a loop has to be performed corresponding to the number of error terms one has decided to draw in the calibration step (a decision that has to be taken beforehand as this is something that only has to be done once and remains fixed until an updated dataset becomes available) in order to determine the choice probabilities.

A final remark that has not been mentioned before is that the procedures outlined above require that MIMOSIS be run several times corresponding to the number of choices in the discrete choice set and that a separate evaluation module that takes into account labour supply responses can only be run once all the results (disposable incomes, taxes, social insurance contributions, etc.) for each of the possible choices

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[1] Of course, it is always possible to estimate several labour supply models with different underlying utility functions, store the estimates in separate input files and have the user decide what model to use. This option presupposes a more profound knowledge of labour supply models on the part of the user however.

have been calculated. Running MIMOSIS several times can easily be automated in some outside loop.

3.1 EXAMPLE OF CALCULATION IN PSEUDO-CODE

In this section we briefly describe the procedure followed to integrate labour supply responses in a policy simulation using pseudo-code. We would like to stress that what follows is no usable code in that it is not written in any one particular programming language. It rather serves as a sequence of logical steps that need to be taken into consideration when programming labour supply reactions.

3.1.1 without calibration

```
read in coefficients of utility function in matrix

initialize denominator in (4): U_sum=0

foreach individual do

        foreach hour point x in the choice set do

                calculate utility level U_x

                U_sum=U_sum+U_x

        end do

        foreach hour point x in the choice set do

                !! probability of choosing x hours of work

                p_x=exp(U_x)/exp(U_sum)

        end do

end do

!! example: calculate total taxes

!! initialize total taxes

tot_tax=0

foreach individual do

        foreach hour point x in the choice set do

                tot_tax=tot_tax + p_x*tax_x*weight
```

```
        !! tax_x is the amount of tax revenue collected from this individual when
        !! working x hours per week

    end do

end do
```

These steps should be performed once for the baseline and once for every reform situation. The aggregate total in baseline and reform can then be compared to assess the effect of incorporating labour supply reactions in the analysis.

3.1.2 with calibration

```
read coefficients of utility function in matrix

read error terms from calibration step in matrix --e.g. 100 per individual

foreach individual do

    foreach hour point x in the choice set do

        !! initialize variable that counts number of times x is optimal

        count_x=0

    end do

    foreach error term do

        foreach hour point x in the choice set do

            calculate the utility level U_x+error term (=V_x)

            put the results in a vector that has as many elements as there are
            hours options, i.e. first element corresponds to utility level when
            working 0 hours, etc.

        end do

        !! determine the maximum value in the vector

        max_value=max(V)

        foreach hour point x in the choice set do

            if max_value=V_x then count_x=count_x+1

        end loop

        (ending the loop here excludes possible ties and implicitly assumes that
        in the case of ties the lowest possible amount of hours will be worked)
```

end do

end do

foreach hour point x in the choice set do end **do**

!! probability of choosing x hours of work per week

p_x=count_x/(#error terms)

end do

end do

The calculation of the labour supply effects on government revenue is then identical to that in the case without calibration.

It should be noted that the above example is in the case of singles. In the case of couples the loops would be a little more involved as one would loop over all possible hour combinations, e.g.:

foreach hour point x in the choice set **do**

foreach hour point y in the choice set **do**

calculated the utility level **U_x_y** of the combination male x hours and female y hours

end do

end do

Probabilities would then also be for a combination of hours, i.e. p_x_y. If the choice set contains 12 discrete hour points for example, the total number of possible combinations for couples would then be 144.

References

Decoster, A. , De Swerdt, K. and Van Camp G. (2010), *Modelling labour supply and policy reform in the Belgian tax benefit model MIMOSIS*, Report for FPS Social Security, Part 2.

McFadden, D., (1974), Conditional logit analysis of qualitative choice behaviour, in Zarembka, P., ed., *Frontiers in Econometrics*, Academic Press, New York.