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The Relativity of Decreasing Inequality Between Countries

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We study the evolution of population-weighted between-country inequality in the period 1980–2009. Whereas previous studies almost exclusively focused on relative inequality measures, we consider relative, absolute and intermediate versions of the Lorenz dominance criterion and of the S-Gini and generalized entropy classes of inequality measures. The analysis yields robust evidence for increasing absolute inequality. Moreover, this conclusion is preserved for intermediate views substantially in the direction of the relative view. In contrast, robust evidence for decreasing inequality—be it relative, absolute or intermediate—is virtually absent. These findings challenge the widely accepted claim of decreasing between-country inequality.

INTRODUCTION

A substantive empirical literature on the evolution of global income inequality has emerged in recent years.[1] The focus has been on the past three to four decades, a period of intense globalization. The change of inequality during this period at least in part reveals how the growth generated by globalization has been divided among the world's citizens. Consequently, the question of how global inequality evolved is highly ideologically charged. The academic debate on the question is vigorous and has not yielded a definitive answer.

Nevertheless, two facts are broadly agreed upon across studies. First, inequality *within* most countries is increasing, notably in highly populated countries such as China and the USA. Second, inequality *between* countries—with each world citizen assigned the per capita income of his country—is decreasing, as several poorer countries have experienced growth rates far exceeding those of richer countries. To come to a conclusion on overall global inequality, the relative sizes of these two opposing trends must be gauged.[2] The different studies have relied on widely varying methodologies to approximate the required—missing—global income distribution, which explains the contradictory conclusions (Anand and Segal 2008).

While different studies disagree on how to approximate the missing data, they are in striking agreement on the inequality measures to be used. The applied inequality measures are almost exclusively of the relative kind, typical examples being the Gini and Theil measures. According to a relative measure, inequality remains invariant if all incomes grow in the same proportion. However, as remarked earlier by Ravallion (2003) and Atkinson and Brandolini (2004, 2010), this exclusive focus on the relative inequality view is unduly restrictive. The theoretical literature on inequality measurement has discussed the alternative absolute and intermediate inequality views—proclaiming unchanged inequality under equal absolute growth, or under a combination of equal proportional and equal absolute growth, respectively—from the very outset (Dalton 1920; Kolm 1969, 1976a, b). The prevailing appreciation in the literature is that one cannot argue conclusively in favour of one specific inequality invariance view using positive arguments only.[3] This appreciation is echoed in questionnaire studies, showing that respondents hold diverse invariance views (Amiel and Cowell 1999a, b; Ballano and Ruiz-Castillo 1993; Harrison and Seidl 1994). In sum, a satisfactory study of the evolution of inequality cannot ignore these alternative—*a priori* equally relevant—invariance views.

In this paper, we study the evolution of inequality using absolute and intermediate measures in addition to relative measures. Our focus is on inequality between countries in the period 1980–2009. A cursory glance at the relevant data already reveals that broadening the perspective beyond the relative view may profoundly affect the conclusions on inequality. Figure 1 shows average yearly growth in GDP per capita between 1980 and 2009 for each of the five quintiles of the 1980 between-country income distribution.[4] The left axis presents growth in relative terms. The yearly growth rate was 8.8% in the bottom quintile, while it was 1.6% in the top quintile. The relative gap between the two quintiles has narrowed significantly—an observation that is supportive of the popular claim of decreasing inequality. Now consider the right axis, which presents growth in absolute terms. The comparison between the bottom and the top quintile is now radically different. Income per capita in the top quintile increased by $431 per year, while that in the bottom quintile increased only by $192 per year—a considerable widening of the absolute income gap. Clearly, a shift from the relative view to the absolute view (or to intermediate views) will cast a different light on the evolution of inequality.

Our objective is to make explicit the dependence of the empirical conclusions on the particular invariance view taken. In doing so, we focus on robust results holding for wide classes of inequality measures. We consider extensions of the Lorenz dominance criterion and of the S-Gini and generalized entropy classes of inequality measures, encompassing the relative, absolute and intermediate views. Our analysis is complementary to that of Atkinson and Brandolini (2004, 2010), who focus on specific (relative, absolute and intermediate) measures of inequality.

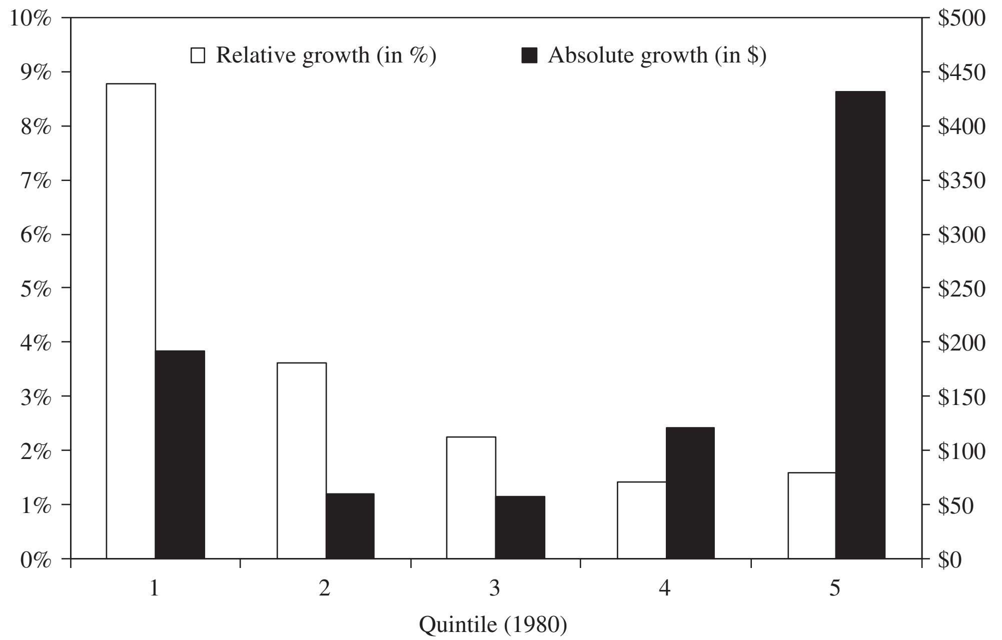

FIGURE 1. Income growth between 1980 and 2009.

Our results challenge the widely accepted claim of decreasing between-country inequality. In fact, the extension of the Lorenz dominance criterion—applied to all 435 pairs of income distributions in the period 1980–2009—provides strong evidence for increasing absolute inequality. This conclusion is preserved for intermediate views substantially in the direction of the relative view. On the other hand, the Lorenz dominance analysis yields virtually no evidence for decreasing relative, absolute or intermediate inequality. More insights can be gained by considering the extensions of the S-Gini and generalized entropy classes of inequality measures. Oft-used relative members of these classes, such as the Gini and Theil measures, indicate decreasing inequality between 1980 and 2009. But other relative members of the classes show increasing inequality, in particular those with an especially high or low sensitivity to the bottom of the income distribution. Absolute members in the two classes—and also intermediate members going well in the direction of the relative view—indicate increasing inequality irrespective of the degree of bottom-sensitivity.

The paper proceeds as follows. In the next section, we provide a generic definition of intermediate inequality: inequality is unchanged if an increase in total income consists for α% in an equal proportional increase and for $(1-\alpha)$% in an equal absolute increase. The parameter α quantifies the position of an intermediate view between the polar absolute ($\alpha = 0$) and relative ($\alpha = 1$) cases. This generic definition covers the specific intermediate concepts proposed by Besley and Preston (1988), Bossert and Pfingsten (1990), Krtscha (1994), Pfingsten (1987), Yoshida (2005), Zheng (2004) and Zoli (2003). In Section II, we extend the Lorenz dominance criterion using this definition of intermediate inequality. We check for Lorenz dominance for each pair of income distributions in the period 1980–2009 and for each value of α between 0 and 1. In Section III, we extend the S-Gini and generalized entropy classes of inequality measures using our definition of intermediate inequality. We identify the combinations of parameter values—the first parameter being α, the second parameter being a measure of bottom-sensitivity—corresponding to each of the competing inequality judgments. Section IV concludes.

I. Absolute, Relative and Intermediate Views

An income distribution is a vector $x = (x_1,x_2,\ldots,x_n)$, where x_i is a positive real number denoting the income of individual $i = 1,2,\ldots,n$. Incomes are ordered such that $x_1 \leq x_2 \leq \cdots \leq x_n$. The set X collects all income distributions. The arithmetic mean of x is denoted by $\overline{x}$. We write 1_n for the n-dimensional vector in which each component is equal to 1.

An inequality measure is a continuous function $I : X \rightarrow \mathbb{R}$ that satisfies the transfer principle. This principle says that with total income fixed, an order-preserving richer-to-poorer transfer decreases inequality. Formally, for each income distribution $(x_1,\ldots,x_i,\ldots,x_j,\ldots,x_n)$ in X and each positive real number δ, if $x_i < x_i + \delta \leq x_j - \delta < x_j$, then $I(x_1,\ldots,x_i,\ldots,x_j,\ldots,x_n) > I(x_1,\ldots,x_i + \delta,\ldots,x_j - \delta,\ldots,x_n)$.

Invariance views extend inequality comparisons to income distributions with different total incomes. An invariance view expresses how a change in total income has to be distributed over the individuals such that inequality remains invariant. Suppose that the initial income distribution is x and that total income grows by $n\lambda$, increasing mean income from $\overline{x}$ to $\overline{x} + \lambda$. According to the relative invariance view, inequality remains the same if every income increases by the same proportion $(\overline{x} + \lambda)/\overline{x}$. That is, income distribution $[(\overline{x} + \lambda)/\overline{x}]x$ exhibits the same inequality as x. According to the absolute invariance view, inequality remains the same if every income increases by the same absolute amount λ. That is, income distribution $x + \lambda 1_n$ exhibits the same inequality as x.

An intermediate invariance view takes a position in between the relative and absolute views. According to an intermediate view—as we define it—inequality remains the same if every income increases by a weighted average of an equal proportional increase and an equal absolute increase. Formally, for each income distribution x and for each amount of per capita growth $\lambda > 0$, there is an α in the interval [0,1] such that income distribution

$$(1) \qquad x' = \alpha \frac{\overline{x} + \lambda}{\overline{x}} x + (1 - \alpha)(x + \lambda 1_n)$$

is equally unequal as x. Note that this definition allows that an intermediate view specifies a different α for each initial income distribution x and each amount of per capita growth λ. In the notation, we suppress this dependency and write α instead of $\alpha(x,\lambda)$. The absolute view corresponds to a constant α equal to 0, the relative view to a constant α equal to 1. For a given income distribution x and growth amount λ, the higher the value of α, the further the corresponding intermediate view is from the absolute view and the closer it is to the relative view. Because α reflects the degree of relativeness for a given x and λ, we will interpret it as a *local* measure of the degree of relativeness.

Table 1 presents an example. The initial income distribution is x = (100,200,300,400). A per capita increase in income of $\lambda = 250$ doubles mean income from $\overline{x} = 250$ to $\overline{x} = 500$. The first row of the table corresponds to the absolute view, with each individual receiving the same absolute increase in income of 250. The final row corresponds to the relative view, with each income being doubled. Each row in between shows the income distribution x', as defined in equation (1), that exhibits the same inequality as x according to an α between 0 and 1. The value of α neatly describes, for a given x and λ, the precise position of an intermediate view between the absolute and relative views. For example, if $\alpha = 0.8$, then each individual's income increases by 80% of the same proportional growth (0.8 times 100%) plus 20% of the same absolute growth (0.2 times 250).

Our definition of the intermediate view is generic, as it leaves open how α varies with the initial income distribution and the amount of per capita growth. The definition covers most invariance concepts proposed in the literature.[5] That is, each such invariance concept specifies, for each income distribution x and each amount of per capita growth λ,

TABLE 1
INTERMEDIATE VIEWS AS WEIGHTED AVERAGES OF THE RELATIVE AND ABSOLUTE VIEWS

Inequality view	α	x'	μ	v	η
Absolute	0	(350, 450, 550, 650)	0	$+\infty$	0
Intermediate	0.1	(335, 445, 555, 665)	0.00044	2250	0.138
Intermediate	0.2	(320, 440, 560, 680)	0.00100	1000	0.263
Intermediate	0.5	(275, 425, 575, 725)	0.00398	250	0.585
Intermediate	0.8	(230, 410, 590, 770)	0.01575	62.5	0.848
Intermediate	0.9	(215, 405, 595, 785)	0.03475	27.7	0.926
Relative	1	(200, 400, 600, 800)	1	0	1

Given income distribution x = (100,200,300,400), how should an extra amount $4\times250(=4\times\lambda)$ be divided—resulting in income distribution x'—such that inequality is unaffected?

a value of α such that income distribution x' in equation (1) is equally unequal as x. In the concept of Pfingsten (1987), further explored by Bossert and Pfingsten (1990), $\alpha = \mu\bar{x}/(\mu\bar{x} + 1 - \mu)$ with μ a constant in the interval [0,1]. In the concept of Besley and Preston (1988), which is based on that of Kolm (1976b), $\alpha = \bar{x}/(\bar{x} + v)$, with v a constant equal to or greater than 0. In a concept defined by Zoli (2003) and Zheng (2004), α is a constant.[6] Finally, in the concept of Yoshida (2005),

$$\alpha = \frac{\frac{(\bar{x}+\lambda)^{\eta}}{\bar{x})} - 1}{\frac{\bar{x}+\lambda}{\bar{x}} - 1},$$

with η a constant in the interval [0,1].[7] The choice of a specific parameter value, μ, v or η, determines a unique α for each x and λ. The final three columns of Table 1 present, for the alternative invariance concepts, the parameter value that corresponds to each given value of α.

In the empirical analysis of the following sections, we do not *a priori* select a specific invariance view. Instead, we check how each pair of income distributions compares for *all* values of α between 0 and 1. Hence the empirical analysis encompasses all of the above invariance concepts proposed in the literature. This will give a complete picture of how inequality judgments depend on the chosen invariance view.

II. Lorenz Dominance

We consider first the Lorenz dominance criterion. To simplify notation, we focus on comparisons of income distributions with the same population size—the extension to different population sizes is straightforward.[8]

For two income distributions x and y with equal means, x is said to Lorenz dominate y if

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i \quad \text{for each } k = 1, 2, \ldots, n-1,$$

with at least one inequality holding strictly. We have defined inequality measures as satisfying the transfer principle. It is well known that x Lorenz dominates y if and only if all inequality measures agree that x is strictly less unequal than y (Moyes 1999).

Now take two income distributions x and y that do not have equal means. Suppose that $\bar{x} < \bar{y}$. To apply the above Lorenz dominance criterion, we need a prior step to equalize the means. It is here that the particular invariance view comes into play. Suppose that α is the local degree of relativeness that we want to impose. Then the income distribution

$$x(\alpha) = \alpha\frac{\bar{y}}{\bar{x}}x + (1-\alpha)\big(x + (\bar{y} - \bar{x})1_n\big) \tag{2}$$

is equally unequal as x, but has a mean equal to that of y. Because the income distributions $x(\alpha)$ and y have equal means, the Lorenz criterion can be used to compare them. This procedure leads to the following α-Lorenz dominance criterion.

Definition 1. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. Let $x(\alpha)$ be defined as in equation (2), with α in the interval [0,1]. We say that x α-Lorenz dominates y if $x(\alpha)$ Lorenz dominates y. We say that y α-Lorenz dominates x if y Lorenz dominates $x(\alpha)$.[9]

Note that α-Lorenz dominance of x over y is equivalent to agreement of all inequality measures with a local relativeness of α—that is, all measures according to which x is equally unequal as $x(\alpha)$—that x is less unequal than y. Furthermore, 1-Lorenz dominance and 0-Lorenz dominance coincide with relative and absolute Lorenz dominance, respectively.[10]

Stronger judgments may be deduced from the fact that x α-Lorenz dominates y, depending on whether x or y has the higher mean. Consider again the example of Table 1, with income distribution $x = (100,200,300,400)$. Let $y = (220,450,580,750)$, an income distribution with twice the mean income of x. First, as is easily checked, x 0.5-Lorenz dominates y since $x(0.5) = (275,425,575,725)$ Lorenz dominates y. Importantly, x also α-Lorenz dominates y for all values of α smaller than 0.5—this is readily verified for the particular cases where α equals 0, 0.1 or 0.2 in Table 1. That is, all inequality measures with a local relativeness between 0 (absolute) and 0.5 agree that inequality has increased in the transition from x to y. Second, y 0.9-Lorenz dominates x since y Lorenz dominates $x(0.9) = (215,405,595,785)$. Again, y also α-Lorenz dominates x for all values of α greater than 0.9—this is easy to check for the particular value $\alpha = 1$ in Table 1. In other words, all inequality measures with a local relativeness between 0.9 and 1 (relative) agree that inequality has decreased in the transition from x to y. Note that neither $x(0.8)$ Lorenz dominates y, nor y Lorenz dominates $x(0.8)$.[11] The following result generalizes the logic of this example.[12]

Proposition 1. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. If x $\widehat{\alpha}$-Lorenz dominates y, then x α-Lorenz dominates y for all $\alpha < \widehat{\alpha}$. If y $\widetilde{\alpha}$-Lorenz dominates x, then y α-Lorenz dominates x for all $\alpha > \widetilde{\alpha}$.

Proof. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. Let x $\widehat{\alpha}$-Lorenz dominate y. That is, we have

$$\sum_{i=1}^{k} \widehat{\alpha}\frac{\overline{y}}{\overline{x}}x_i + (1-\widehat{\alpha})(x_i + \overline{y} - \overline{x}) \geq \sum_{i=1}^{k} y_i \quad \text{for each} \quad k = 1, 2 \ldots, n\text{–}1,$$

with at least one inequality holding strictly. Let $\alpha < \widehat{\alpha}$. We have to show that x α-Lorenz dominates y. It suffices to show that for each $k = 1, 2, \ldots, n-1$,

$$(3) \qquad \sum_{i=1}^{k} \alpha\frac{\overline{y}}{\overline{x}}x_i + (1-\alpha)(x_i + \overline{y} - \overline{x}) \geq \sum_{i=1}^{k} \widehat{\alpha}\frac{\overline{y}}{\overline{x}}x_i + (1-\widehat{\alpha})(x_i + \overline{y} - \overline{x}).$$

Rewrite equation (3) as

$$(\widehat{\alpha}-\alpha)\sum_{i=1}^{k}(x_i+\overline{y}-\overline{x})\geq(\widehat{\alpha}-\alpha)\sum_{i=1}^{k}\frac{\overline{y}}{\overline{x}}x_i.$$

Using the fact that $\alpha < \widehat{\alpha}$ and letting $\overline{y} = (1 + \lambda)\overline{x}$, this becomes

$$\sum_{i=1}^{k}(x_i+\lambda\overline{x})\geq\sum_{i=1}^{k}(x_i+\lambda x_i),$$

or

$$\lambda k\overline{x}\geq\lambda\sum_{i=1}^{k}x_i.$$

Since $\lambda > 0$, this indeed holds for each $k = 1,2,\ldots,n - 1$. We omit the similar proof for the case where y $\widetilde{\alpha}$-Lorenz dominates x.

We now turn to the analysis of inequality between countries in the period 1980–2009. The between-country income distribution consists of the total number of individuals of the countries in our sample, with each individual assigned the GDP per capita of his country as income. See the Appendix for more details on the data and testing procedure. For each of the total of 435 pairwise comparisons of income distributions between 1980 and 2009, we check α-Lorenz dominance for all values of α in the interval [0,1]. Proposition 1 considerably simplifies this: for each pair of income distributions x and y with $\overline{x} \leq \overline{y}$, we need only present the maximal α for which x α-Lorenz dominates y, and the minimal α for which y α-Lorenz dominates x. We start with the former.

Table 2 presents the years in order of increasing mean income. Mean world GDP per capita increased in almost every year between 1980 and 2009, the exceptions being 1982 (decrease of 1.2%) and 2009 (decrease of 1.5%). Each cell of the table gives the maximal value of α for which the row year with lower mean income α-Lorenz dominates the column year with higher mean income. We report '–' if there is no value of α such that the row year α-Lorenz dominates the column year.

Let us consider the comparison of the income distributions of 1980 and 2005, a typical case with increasing mean income through time. The critical α reported is 0.46. This means that the income distribution of 1980 α-Lorenz dominates the income distribution of 2005 for $\alpha = 0.46$ and—using Proposition 1—for all values of α between 0 and 0.46. In other words, all inequality measures with a local relativeness between 0 (absolute) and 0.46 agree that inequality has increased between 1980 and 2005.

Table 2 provides substantial evidence for increasing absolute inequality between countries through time. There exists a critical α in 216 out of the total of 435 comparisons. With the exception of two comparisons—those of 2009 with 2007 and 2008—these all involve Lorenz dominance of an earlier year over a later year. Furthermore, the verdict of increasing inequality through time is supported by intermediate views that go well in the direction of the relative view. For the 214 critical values of α corresponding to increasing inequality, the median value is 0.48 and the mean value is 0.45.

TABLE 2
MAXIMAL α FOR WHICH THE ROW YEAR α-LORENZ DOMINATES THE COLUMN YEAR

	1982	1983	1980	1981	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2009	2007	2008
1982		–	–	–	–	0.45	0.48	0.47	0.52	0.58	0.61	0.56	0.53	0.53	0.55	0.54	0.54	0.56	0.54	0.54	0.57	0.54	0.52	0.50	0.50	0.49	0.48	–	0.46	0.44
1983			–	–	0.59	–	0.55	0.52	0.56	0.62	0.65	0.59	0.55	0.55	0.58	0.56	0.56	0.58	0.56	0.56	0.58	0.55	0.53	0.51	0.50	0.50	0.49	–	0.47	0.44
1980				–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.42	0.45	0.45	0.46	0.45	–	0.46	–	–	–	–
1981					–	–	–	–	–	–	–	–	–	–	–	0.06	0.18	0.31	0.41	0.47	0.50	0.50	0.49	0.47	0.48	0.46	0.46	0.32	0.44	0.41
1984						–	–	–	–	0.63	0.31	–	0.55	0.33	–	–	–	–	–	0.50	0.20	–	0.51	0.49	–	–	–	–	–	–
1985							0.59	0.52	0.59	0.68	0.72	0.62	0.57	0.54	0.54	0.52	0.53	0.55	0.53	0.53	0.56	0.53	0.50	0.48	0.49	0.47	0.46	–	0.44	0.41
1986								0.45	–	0.64	0.65	–	0.56	0.51	0.10	–	–	–	0.36	0.52	0.55	0.52	0.49	0.47	–	0.46	0.12	–	–	–
1987									–	0.59	0.62	–	0.59	0.49	–	–	–	–	–	0.51	0.54	0.51	0.48	0.46	–	0.45	–	–	–	–
1988										0.85	0.80	0.23	0.54	0.41	0.45	0.44	–	–	0.48	0.48	0.53	0.49	0.46	0.44	–	0.44	0.43	–	0.41	–
1989											–	–	0.13	–	–	–	–	–	–	–	–	–	0.43	0.40	–	–	–	–	–	–
1990												–	–	–	–	–	–	–	–	–	0.42	–	0.41	0.39	–	0.01	–	–	–	–
1991													0.08	–	–	–	–	–	0.0–	0.43	0.49	0.44	0.42	0.40	–	0.41	0.21	–	–	–
1992														–	–	–	–	–	–	–	–	–	0.44	0.20	–	–	–	–	–	–
1993															–	–	–	–	–	0.34	0.21	–	0.49	0.45	–	–	–	–	–	–
1994																–	–	–	0.50	0.51	0.58	0.51	0.47	0.44	–	0.45	0.05	–	0.00	–
1995																	–	–	0.54	0.54	0.61	0.53	0.49	0.45	–	0.46	–	–	–	–
1996																		–	0.53	0.53	0.63	0.53	0.48	0.43	–	0.45	0.44	–	0.41	0.10
1997																			0.09	0.37	0.59	0.45	0.40	0.36	–	0.41	0.40	–	0.38	0.00
1998																				0.54	0.71	0.52	0.43	0.36	–	0.40	–	–	0.10	–
1999																					0.00	–	0.36	0.29	–	–	–	–	–	–
2000																						–	–	–	–	0.10	–	–	–	–
2001																							–	0.06	–	–	–	–	–	–
2002																								–	–	–	–	–	–	–
2003																									–	–	–	–	–	–
2004																										0.33	0.33	–	0.27	0.00
2005																											–	–	–	–
2006																												–	–	–
2009																													0.80	0.98
2007																														–
2008																														

Of course, the finding of increasing inequality for a lower range of values of α (those reported in Table 2) can go together with the opposite finding of decreasing inequality for an upper range of values of α. The example above Proposition 1 discusses such a case. Therefore the second part of the analysis consists in computing the minimal values of α for which a year with a higher mean income α-Lorenz dominates a year with a lower mean income. However, we do not present the table with these critical values of α. The reason is simple: for none of the 435 pairwise comparisons does there exist a value of α such that a year with a higher mean income α-Lorenz dominates a year with a lower mean income. The table would be empty. Hence the Lorenz analysis provides no support—the two comparisons involving the year 2009 excepted—for decreasing inequality between countries through time.

It is striking that there is not a single instance of relative Lorenz dominance among the total of 435 comparisons. That is, for no comparison do all relative inequality measures agree on the direction of change in inequality. This suggests that the common finding in the literature of decreasing relative inequality between countries relies on the particular subset of relative inequality measures used. In the next section we take a closer look at the conclusions for specific inequality measures.

III. Inequality Measures

We focus on the S-Gini and generalized entropy classes of inequality measures, which include most of the measures used in practice. First, we define the relative cases of the two classes, then we consider extensions that also include absolute and intermediate measures.

We first consider the class of S-Gini inequality measures (Donaldson and Weymark 1980, 1983). Recall that i is the rank position of individual i in the income distribution (since incomes are ordered such that $x_1 \leq x_2 \leq \cdots \leq x_n$). For the relative case, we have

$$(4)\qquad G_\rho(x) = 1 - \sum_{i=1}^{n}\left[\left(\frac{n-i+1}{n}\right)^{\rho} - \left(\frac{n-i}{n}\right)^{\rho}\right]\frac{x_i}{\bar{x}}, \quad \rho \geq 1.$$

Next, we consider the class of generalized entropy inequality measures (Bourguignon 1979; Cowell 1980; Cowell and Kuga 1981a, b; Shorrocks, 1980, 1984). For the relative case, we have

$$(5)\qquad E_\gamma(x) = \begin{cases} \frac{1}{n(\gamma^2-\gamma)}\sum_{i=1}^{n}\left[\left(\frac{x_i}{\bar{x}}\right)^{\gamma} - 1\right] & \text{for } \gamma \neq 0, 1, \\ \frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{\bar{x}}{x_i}\right) & \text{for } \gamma = 0, \\ \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\bar{x}}\right)\ln\left(\frac{x_i}{\bar{x}}\right) & \text{for } \gamma = 1. \end{cases}$$

The parameter ρ in the S-Gini class and the parameter γ in the generalized entropy class both measure the degree of bottom-sensitivity. The higher the value of ρ, or the lower the value of γ, the more weight is given to transfers at the bottom of the income distribution relative to transfers at the top.[13] Setting $\rho = 2$ in the S-Gini class gives the Gini measure. For the generalized entropy class, setting $\gamma < 1$ gives measures that are ordinally equivalent to the class of measures proposed by Atkinson (1970), setting $\gamma = 0$ gives the mean logarithmic deviation, and setting $\gamma = 1$ gives the Theil measure.

The classes in equations (4) and (5) are relative. We extend these classes to include also absolute and intermediate measures using the approach of the previous section.

Definition 2. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. Let $x(\alpha)$ be defined as in equation (2), with α in the interval [0,1]. We say that x is at least as unequal as y according to the (α,ρ)-S-Gini inequality measure if $G_\rho(x(\alpha)) \geq G_\rho(y)$. We say that y is at least as unequal as x according to the (α,ρ)-S-Gini inequality measure if $G_\rho(y) \geq G_\rho(x(\alpha))$.

Definition 3. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. Let $x(\alpha)$ be defined as in equation (2), with α in the interval [0,1]. We say that x is at least as unequal as y according to the (α,γ)-generalized entropy inequality measure if $E_\gamma(x(\alpha)) \geq E_\gamma(y)$. We say that y is at least as unequal as x according to the (α,γ)-generalized entropy inequality measure if $E_\gamma(y) \geq E_\gamma(x(\alpha))$.

However, it can be shown that the criterion in Definition 3 is not transitive (contrary to the criterion in Definition 2).[14] For this reason, we rely instead on the method of Bossert and Pfingsten (1990) to extend the generalized entropy class.[15]

Definition 4. Let x and y be two income distributions in X such that $\overline{x} \leq \overline{y}$. We say that x is at least as unequal as y according to the Bossert–Pfingsten (α,γ)-generalized entropy inequality measure if

$$E_\gamma\left(x + \frac{1-\mu}{\mu}1_n\right) \geq E_\gamma\left(y + \frac{1-\mu}{\mu}1_n\right),$$

with $\alpha = \mu\overline{x}/(\mu\overline{x} + 1 - \mu)$. We say that y is at least as unequal as x according to the Bossert–Pfingsten (α,γ)-generalized entropy inequality measure if

$$E_\gamma\left(y + \frac{1-\mu}{\mu}1_n\right) \geq E_\gamma\left(x + \frac{1-\mu}{\mu}1_n\right),$$

with $\alpha = \mu\overline{x}/(\mu\overline{x} + 1 - \mu)$.[16]

Figure 2 summarizes the empirical results based on the inequality measures in Definitions 2 and 4. The income distribution of the year 2009 is compared with the income distributions of 1980, 1990 and 2000. Each of these three pairwise comparisons is represented by a curve in the two-parameter space of (α,ρ) for the S-Gini class and (α,γ) for the generalized entropy class. The curve depicts the parameter combinations for which inequality remains unchanged between the two years under comparison. For parameter combinations to the right of the curve, inequality has decreased through time, while for parameter combinations to the left, inequality has increased.

We first consider the relative inequality measures in Figure 2. As mentioned before, the literature has focused on just a handful of relative measures. Particularly popular are the Gini measure, corresponding to the point $(\alpha,\rho) = (1,2)$ in panel (a) of Figure 2, and the mean logarithmic deviation and the Theil measure, corresponding to the points $(\alpha,\gamma) = (1,0)$ and $(\alpha,\gamma) = (1,1)$ in panel (b) of the figure, respectively. These popular measures lie comfortably in the area of decreasing inequality for each of the three pairwise comparisons. However, neither for the S-Gini class, nor for the generalized entropy class, do the relative members agree unanimously on decreased inequality. Increased inequality between 1980 and 2009 is concluded by the relative S-Gini inequality measures for high degrees of bottom-sensitivity ($\rho > 32.4$) and by the relative generalized entropy measures for high ($\gamma < -6.4$) or low ($\gamma > 15.2$) degrees of bottom-sensitivity.[17] Of course, these degrees of bottom-sensitivity are extreme, which shows that a focus on the very highest incomes or on the very lowest incomes is required in order to conclude increased relative inequality.

Next, we consider the absolute and intermediate inequality measures. Figure 2 shows that there exist values of $\alpha < 1$ such that all corresponding members of the S-Gini class or the generalized entropy class agree on increased inequality. Members of the S-Gini class corresponding to values of α from 0 to 0.53 unanimously agree that inequality has increased between 1980 and 2009. For the generalized entropy class, this range is from 0 to 0.49. Note that the income distributions of 1980 and 2009 are Lorenz incomparable (see Table 2), meaning that there is no value of α such that all corresponding inequality measures agree on the direction of the inequality change. Hence the finding of a unanimous verdict of increased inequality for the two narrower classes of inequality measures is not trivial.

IV. Conclusion

We assessed the evolution of relative, absolute and intermediate inequality between countries in the period 1980–2009. Our findings strongly challenge the widely accepted claim of decreasing inequality. Indeed, using the Lorenz criterion, we found increasing absolute inequality through time in 214 out of the total of 216 Lorenz comparable pairs of income distributions in the period. In these 214 cases, the conclusion of increasing inequality is preserved for intermediate views up to about halfway to the relative view. In striking contrast, the Lorenz dominance analysis does not yield a single instance of decreasing relative inequality among the total of 435 pairwise comparisons. While popular relative members of the S-Gini and generalized entropy classes—the Gini and Theil measures and the mean logarithmic deviation—indicate decreasing inequality between 1980 and 2009, this finding does not extend to all relative inequality measures in these classes. On the other hand, the absolute members of these classes, as well as intermediate members up to about halfway to the relative view, agree unanimously on increasing inequality in the period.

One may wonder what—if anything—can be concluded about the evolution of overall global inequality, that is, the inequality between all citizens of the world. To tackle this question, we would need to establish how absolute and intermediate within-country inequality changed, and add this to our findings on between-country inequality. This is beyond the scope of this paper. Nonetheless, tentative insights may be obtained by relying on the widespread claim of increasing relative within-country inequality. The combination of increasing relative inequality and increasing income within countries strongly suggests that absolute and intermediate within-country inequality have increased as well. This would mean that for decomposable inequality measures, our findings extend to global inequality: global inequality has increased for the absolute view and for intermediate

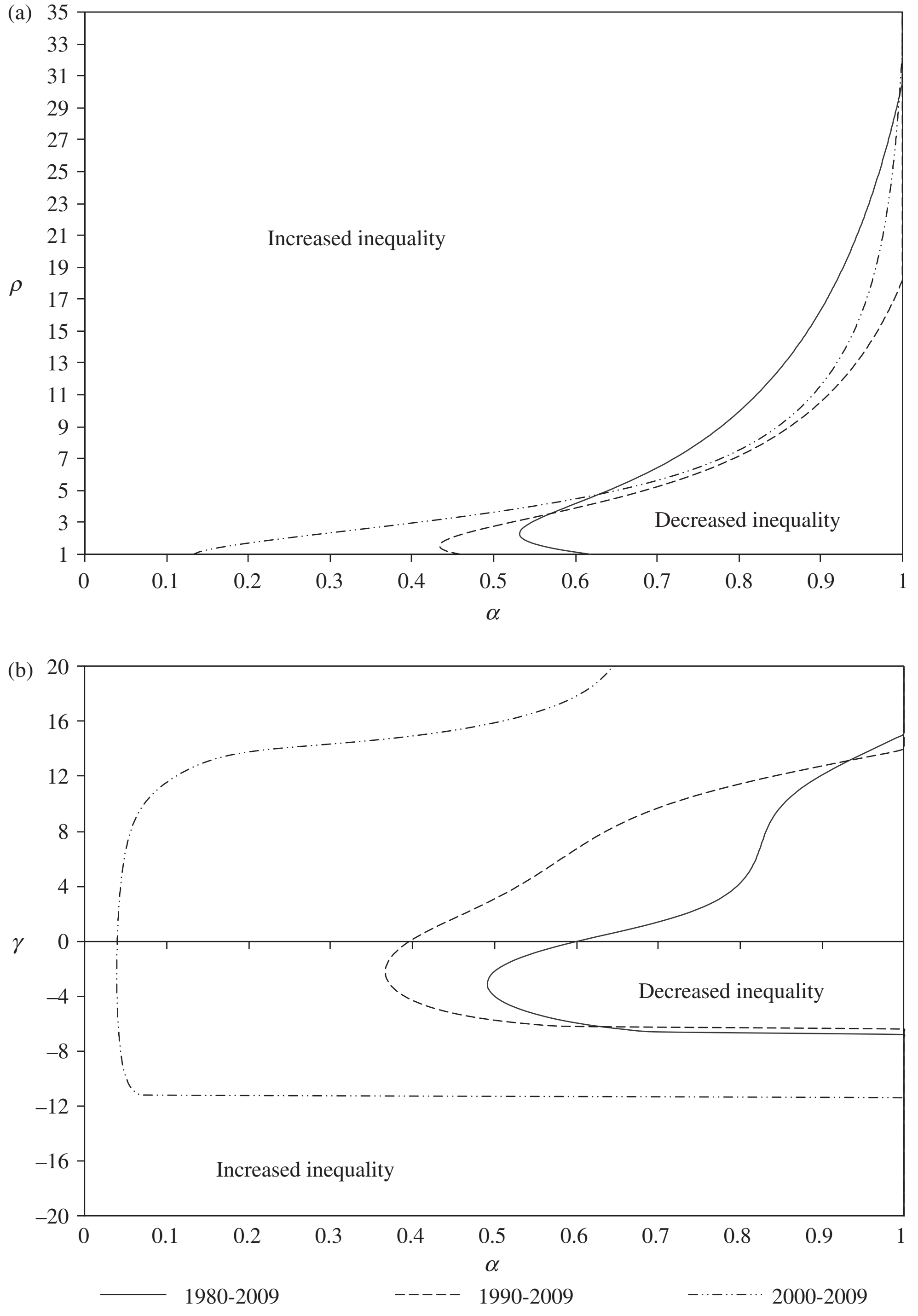

FIGURE 2. Inequality comparisons (a) for the S-Gini class in the (α,ρ) space and (b) for the generalized entropy class in the (α,γ) space.

views sufficiently close to the absolute view. Testing this hypothesis ultimately requires world-scale micro-level data. We leave this to future research.[18]

Our analysis has shown that empirical conclusions on the evolution of between-country inequality depend crucially on the chosen invariance view. As one moves from the relative view, predominating in the empirical literature, to the absolute view, the conventional wisdom of decreasing inequality is completely reversed. This underlines the importance of explicitly discussing and justifying the properties of the employed measurement apparatus. Rather than being mere technicalities, these properties embody the meaning of inequality.

APPENDIX: DATA DESCRIPTION AND TESTING PROCEDURE

We use the World Development Indicators (WDI) dataset, downloaded from the website of the World Bank at http://databank.worldbank.org/data/home.aspx in July 2010. The WDI dataset contains 212 countries. For our analysis, we retain only the countries for which data are available for the entire period 1980–2009. This reduces the dataset to the 115 countries listed in Table A1. The ten most populated absentees are, in order of decreasing population size, Russia, Vietnam,

TABLE A1
COUNTRIES IN THE DATASET

Albania	Dominican Republic	Latvia	Romania
Algeria	Ecuador	Lesotho	Rwanda
Antigua and Barbuda	Egypt	Liberia	Saint Kitts and Nevis
Argentina	El Salvador	Luxembourg	Saint Lucia
Australia	Estonia	Madagascar	Saint Vincent and the Grenadines
Austria	Fiji	Malawi	Saudi Arabia
Bangladesh	Finland	Malaysia	Senegal
Belgium	France	Mali	Seychelles
Benin	Gabon	Mauritania	Sierra Leone
Bolivia	Gambia	Mauritius	Singapore
Botswana	Germany	Mexico	South Africa
Brazil	Ghana	Morocco	Spain
Bulgaria	Greece	Mozambique	Sri Lanka
Burkina Faso	Grenada	Namibia	Sudan
Burundi	Guatemala	Nepal	Swaziland
Cameroon	Honduras	Netherlands	Sweden
Canada	Hungary	New Zealand	Syria
Central African Republic	Iceland	Nicaragua	Thailand
Chad	India	Niger	Togo
Chile	Indonesia	Nigeria	Trinidad and Tobago
China	Iran	Norway	Tunisia
Colombia	Ireland	Pakistan	Turkey
Comoros	Israel	Panama	United Kingdom
Congo	Italy	Papua New Guinea	United States
Costa Rica	Jamaica	Paraguay	Uruguay
Côte d'Ivoire	Japan	Peru	Vanuatu
Democratic Republic of the Congo	Jordan	Philippines	Venezuela
Denmark	Kenya	Portugal	Zambia
Dominica	Kiribati	Republic of Korea	

Ethiopia, Myanmar, Ukraine, Tanzania, Poland, Uganda, Iraq and Afghanistan. Our dataset covers 86.39% of the 2009 population in the full WDI dataset.

We use real GDP per capita, taking 2005 as the reference year for the constant prices. To correct for relative price differences, we rely on the standard PPP factors used in the WDI dataset. These PPP factors are computed by the International Comparison Program of the World Bank (see Deaton 2010 for a critical appraisal). The mean 2009 GDP per capita is \$5393 in the full WDI dataset, whereas it is \$6005 in our dataset.

The results in Table 2 were obtained as follows. Consider a pair of income distributions. First, we transform the income distribution with the lower mean using equation (2). Second, we test for Lorenz dominance using a conservative procedure that compares the ordinates of the Lorenz curves on each corner of the two empirical cumulative distribution functions. We perform this test over a fine grid of values of α to obtain the values reported in Table 2.

The results in Figure 2 are obtained using an algorithm that computes—over a fine grid of ρ and γ values—the values of α for which the relevant inequalities in Definitions 2 and 4 hold with equality. All algorithms were programmed in MATLAB and are available on request.

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NOTES

1. Among others, see Berry *et al.* (1983), Bourguignon and Morrisson (2002), Capéau and Decoster (2005), Chotikapanich *et al.* (1997, 2012), Dowrick and Akmal (2005), Milanovic (2002, 2010), Pinkovskiy and Sala-i-Martin (2009) and Sala-i-Martin (2006). See Anand and Segal (2008) and Milanovic (2005) for surveys.
2. Not all inequality measures can be neatly decomposed into within-country and between-country parts. For instance, the Gini measure features an additional component that measures the overlap between countries' income distributions. See, for example, Milanovic (2005, ch. 3).
3. The relative invariance requirement is sometimes interpreted as an innocent technical property to make inequality comparisons independent of the unit in which incomes are measured. But this interpretation is problematic (e.g. Kolm 1976a, pp. 419–20; Marchant 2008, pp. 694–5). Invariance requirements deal with the normative issue of how to distribute amounts of various sizes in an inequality-neutral way. Hence they reflect an important aspect of the notion of inequality extending beyond the independence of the unit of measurement. In our empirical application, the issue of the unit of measurement is treated coherently by expressing all incomes in PPP US dollars at constant prices.
4. GDP per capita is expressed in 2005 PPP US dollars. The Appendix describes the data and the sample selection.
5. An exception is the concept of Seidl and Pfingsten (1997), including the special case studied by del Río and Ruiz-Castillo (2000). The precise form of the latter concept is determined jointly by the value of a parameter (similar to α) and by a reference income distribution z. The dependence on the reference income distribution z renders the concept of del Río and Ruiz-Castillo inconsistent with equation (1). But their concept does allow an interpretation along the lines of equation (1) for a limited set of pairwise comparisons: those comparisons of income distributions x and y, with $\bar{x} \leq \bar{y}$, for which x happens to coincide with the reference income distribution z. The empirical analysis of del Río and Ruiz-Castillo (2001) deals with such a comparison and exploits this interpretation.
6. Zoli (2003) and Zheng (2004) formulate this concept with the purpose of representing the concept of del Río and Ruiz-Castillo (2000). However, as noted by del Río and Alonso-Villar (2010, footnote 7), the concept of Zoli and Zheng does not coincide with the concept of del Río and Ruiz-Castillo. The latter is not consistent with our generic definition of intermediate inequality because of the dependence on a reference income distribution. Nonetheless, the simple invariance concept of Zoli and Zheng is interesting in its own right.
7. Yoshida (2005) generalizes Krtscha (1994). In the latter concept, $\eta = 0.5$.
8. For our empirical analysis, we extend the Lorenz criterion in Definition 1 using the replication invariance principle. This principle says that replications of the income distribution leave inequality unchanged. If two income distributions have different population sizes, then they can be replicated up to the same population size and compared using Definition 1. Note that the inequality measures in Section III also satisfy the replication invariance principle.

9. We saw in Section I that our definition of intermediate inequality covers several of the invariance concepts proposed in the literature. By consequence, the α-Lorenz criterion is consistent with the Lorenz criteria based on these invariance concepts. Consider as an example the concept of Pfingsten (1987) (and Bossert and Pfingsten 1990). Let x and y be two income distributions such that $\overline{x} < \overline{y}$. The condition for α-Lorenz dominance of x over y is $\sum_{i=1}^{k} \alpha(\overline{y}x_i/\overline{x}) + (1-\alpha)(x_i + \overline{y} - \overline{x}) \geq \sum_{i=1}^{k} y_i$ for each $k = 1,2,\ldots,n-1$, with at least one inequality holding strictly. In the invariance concept of Pfingsten (1987), we have $\alpha = \mu\overline{x}/(\mu\overline{x} + 1 - \mu)$. Substituting this into the condition for α-Lorenz dominance, we obtain the condition for Pfingsten's (1986) Lorenz criterion: $\sum_{i=1}^{k}(x_i - \overline{x})/(\mu\overline{x} + 1 - \mu) \geq \sum_{i=1}^{k}(y_i - \overline{y})/(\mu\overline{y} + 1 - \mu)$.
10. See Moyes (1987) for a comparison of relative and absolute Lorenz dominance. This paper introduced the absolute Lorenz dominance criterion into the literature.
11. To be precise, x α-Lorenz dominates y for all α in the interval [0,0.51], and y α-Lorenz dominates x for all α in the interval [0.87,1]. For other values of α, income distributions x and y are not Lorenz comparable.
12. See del Río and Ruiz-Castillo (2000, pp. 232–3) for a related discussion. Moyes (1992, Proposition 4) provides a version of Proposition 1 covering the relative and absolute cases.
13. With ρ sufficiently high, or γ sufficiently low, the focus is on the smallest income as a fraction of the mean. With γ sufficiently high, the focus is on the highest income as a fraction of the mean.
14. To see this, consider the measure with parameter values $\alpha = 0$ and $\gamma = 1$, and the income distributions $x = (10,30,50)$, $y = (14,22,54)$, $x' = x + (10,10,10) = (20,40,60)$ and $y' = y + (10,10,10) = (24,32,64)$. Since the (0,1)-generalized entropy measure is absolute, income distributions x and x' are equally unequal, and the same goes for y and y'. Because x and y have equal means, Definition 3 demands that we follow the judgment of the relative generalized entropy measure with $\gamma = 1$. The same is true for x' and y'. We have $E_1(x) > E_1(y)$ and $E_1(x') < E_1(y')$, hence x is more unequal than y, and x' is less unequal than y'. We obtain the following cycle: x' is equally unequal as x, x is more unequal than y, y is equally unequal as y', and y' is more unequal than x'.
15. The criterion in Definition 2 is equivalent to the Bossert and Pfingsten (1990, p. 132) extension of the Gini inequality measures. That is, for all income distributions x and y in X, we have that x is at least as unequal as y according to the (α,ρ)-S-Gini inequality measure if and only if $\overline{x}G_\rho(x)/(\mu\overline{x} + 1 - \mu) \geq \overline{y}G_\rho(y)/(\mu\overline{y} + 1 - \mu)$, with $\alpha = \mu\overline{x}/(\mu\overline{x} + 1 - \mu)$ if $\overline{x} \leq \overline{y}$, and $\alpha = \mu\overline{y}/(\mu\overline{y} + 1 - \mu)$ if $\overline{x} \geq \overline{y}$.
16. Cowell (2006) shows how to obtain the absolute members of this class by taking the limit $(1-\mu)/\mu \to +\infty$. This absolute subclass has been characterized axiomatically by Chakravarty and Tyagarupananda (1998, 2009) and Bosmans and Cowell (2010).
17. See Capéau and Decoster (2005) for related findings.
18. Some first insight may be obtained using the approximation of the global income distribution by Sala-i-Martin (2006). The data cover the period 1970–2000 and were downloaded from Xavier Sala-i-Martin's website in December 2008. The Lorenz analysis yields maximal values of α—to be interpreted in the same way as the values in Table 2—in 453 out of the total of 465 pairwise comparisons. With the exception of four comparisons, these all correspond to increasing inequality through time. For the 449 values of α corresponding to increasing inequality, the mean value is 0.85 and the median value is 0.87. Clearly, the support for increasing absolute and intermediate inequality is substantially stronger than that found in Section II. There exist only five minimal values of α, four of which correspond to decreasing inequality through time (these have a mean value of 0.87 and a median value of 0.96). Like the other approximations of the global income distribution in the literature, Sala-i-Martin's approximation unavoidably rests on debatable assumptions—see Anand and Segal (2008) for a critical discussion. Therefore these results must be treated with the necessary caution.

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