Estimating and simulating with a random utility – random opportunity model of job choice

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Abstract

We present the Random Utility/Random Opportunity (RURO) model of job choice (Aaberge, Dagsvik and Strøm, 1995, and Aaberge, Colombino and Strøm, 1999) and report estimation results from an application of a version of that model on Belgian data (EU–SILC 2007). We discuss the effect of education level on the intensity of preference for leisure relative to consumption, on the intensity of job offers, and on the wage offer distribution. Finally, we report simulation results with respect to the impact on labour market participation by letting the male catch up the arrears in educational attainment they currently have with respect to females.

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1 Introduction

The purpose of the present paper is to explain and illustrate how to estimate and simulate with a Random Utility Random Opportunity (henceforth RURO) model of job choice (see amongst others Dagsvik and Strøm, 1992, 2003, and 2006, Aaberge, Dagsvik and Strøm, 1995, Aaberge, Colombino and Strøm, 1999, Dagsvik, Locatelli and Strøm 2006, 2007).[1] These type of models, first developed in an abstract setting by Dagsvik (1994), try to understand individual heterogeneity in choice behaviour as a combined effect of preference differences and differences in opportunities. Contrary to the classical discrete choice models of labour supply (Van Soest, 1995), this approach provides a structural framework for assessing the impact of both, labour supply and labour demand side effects, on labour market participation.

Traditionally, preferences in labour supply models capture the extent to which an individual is willing to trade–off leisure for consumption. It may however deserve recommendation to take into account also other aspects influencing the choice between alternative leisure activities and available jobs, such as social relations involved, challenge of the tasks, security and health, recognition, and societal relevance. But these factors are not easily observable by the analyst. It was one of the contributions of the development of probabilistic choice and random utility models, as developed by respectively Luce (1959) and McFadden (1973), to integrate these additional determinants of preferences as a non–systematic element, affecting the utility obtained from different available alternatives. By non–systematic it is meant that such factors are uncorrelated, and therefore cannot be explained by observable characteristics. Random utility models have been applied to labour supply behaviour (Van Soest, 1995) since. They replaced the traditional continuous choice approach to labour supply behaviour (see Hausman, 1985, for a review of this traditional approach), which faced difficulties in deriving tractable closed form solutions of labour supply functions, in the presence of non–linear budget sets. Indeed, many personal income tax systems, such as *e.g.* a minimum income guarantee associated with a linear earned income tax, create non–convexities in the budget set (the available bundles of consumption and labour time a person can chose from), leading to discontinuities and non–uniqueness of the optimal choice in function of wage variations. These phenomena are more easily treated in a discrete choice set–up, which is the approach taken by both, probabilistic choice and random utility models.

The random utility model is however still limited in scope. Interindividual differences in the availability of alternatives from which a person can choose, are exogenous to the model. Applied to job choice, differences in individual budget sets stem exclusively from wage dif-

[1] Recent overviews of the model and its applications are provided by Aaberge and Colombino (2014), and Dagsvik, Jia, Kornstad, and Thoresen (2014).

ferences and differences in unearned income. In a static model, it is indeed reasonable to assume that unearned income differences are exogenous, and do not depend on individual choices. But in standard random utility models also wages are exogenously fixed individual characteristics, reflecting a person's productive capacities.[2] This is unattractive. Productive capacities can in many cases not be determined appropriately, when considered separately from the specific job in which these capacities are exhibited. Moreover, it is quite unnatural that all available job offers, even when perfectly suited to a person's capacities and skills, would pay the same wage. Besides the question of the meaning and determination of the wage, it is also highly unlikely that persons can completely freely fix the number of hours they will work, due to organisational limitations of the production process and social life.

It is exactly these type of frictions in the choice process which are taken into consideration by RURO models, as an additional factor, next to preferences, to understand choice behaviour. Job offers are considered as packages of wages, labour time regimes, and a number of other attributes (Dagsvik and Strøm, 1992, 2003, Aaberge, Dagsvik and Strøm, 1995, and Aaberge, Colombino and Strøm, 1999). These other attributes (challenge, safety and security, esteem and recognition, appreciation of colleagues, responsibility...) are however difficult to observe, especially as far as it concerns the degree of job satisfaction they can provide to a person. Therefore, the individual specific availability of suitable jobs is thought of as the result of a stochastic process of job offers. The impact of explanatory variables on the intensity with which job offers arrive to a person according to that process, is estimated jointly with individual's preference characteristics. Not only the intensity with which job offers arrive, but also the availability of, according to a person's own judgement, attractive non–market alternatives to spend time, is individually specific. Limited physical abilities might impede someone who likes to walk outside, to do so. Choosing under such circumstances between sitting in front of a liquid crystal screen, reading books, or accepting a job, that person might opt for the latter, while the reverse might happen for someone with similar preferences, but in good physical shape. The RURO model also allows for individual heterogeneity in restrictions on available labour time regimes, even though the effect of these type of restrictions are difficult to identify from the contribution of preferences.

The RURO model is not the only one that embodies restrictions on the choice set into a labour supply model (see, amongst others, Altonji and Paxson 1982, 1992, Van Soest, Woittiez and Kapteyn 1990, Tummers and Woittiez 1991, Dickens and Lundberg 1993, Bloemen 2000,

[2] Some contributions do allow for unobserved wage heterogeneity, see *e.g.* Van Soest, Das and Gong (2002), Löffler *et al.* (2013), and the second model discussed in Dagsvik and Jia (2014). Van Soest (1995) already incorporated the problem of imperfect observation of wages for non–participants in the extended version of his model. Besides, there is an earlier literature accounting for the fact that wages are non–linear in hours (See for example Moffitt 1984). However, none of these treats wages as an object of job choice behaviour.

2008, Ham and Reilly 2002, and Beffy *et al.* 2014). It must be added that the inclusion of dummies for part time and full time work in the discrete choice model of Van Soest (1995), to improve the fit, is in fact a simplified reduced form approach of earlier work with Woittiez and Kapteyn (Van Soest, Woittiez and Kateyn, 1990) which models hours restrictions more explicitly. But the RURO model is the first one that derives these restrictions from an explicit model of a job arrival process, and stresses the individual heterogeneity, both observed and unobserved, of the availability of job offers suitable to the capacities of individual agents.

In the present paper we present estimation results of a version of the RURO model of job choice on Belgian data (EU–SILC 2007).[3] In our data the unemployment rate of lowly educated males is more than 20%, as compared to somewhat more than 4% among highly educated males, and a general unemployment rate of 10%. For females these figures are almost 50% for the lowly educated, 8.5% among highly educated, and almost 24% in general. We illustrate the potential contribution of the different components of the model to explain these figures: the relative intensity of preference for leisure, the intensity of suitable job offers, and the wage offer distribution. Next, we report simulation results assessing the impact on labour market participation of a scenario in which males' education level largely catches up that of females.[4]

In Section 2 we give a self–contained exposition of the RURO model. Section 3 discusses the resulting likelihood function and explains the estimation method. We also devote some time to explain how the estimated model can be used for simulation purposes, both to assess the fit, or to predict the effects of counterfactuals such as tax reforms. Technical issue are relegated to appendices available as supplementary material *on line.* The data are presented in Section 4. Section 5 contains the estimation results. We give graphical representations of the estimated preferences and opportunities side of the estimated model. In Section 6 we investigate the fit of the estimated model, and its behavioural implications. Finally, Section 7 presents the results of the simulation exercise with respect to the effects of a counterfactual educational attainment level. Section 8 concludes.

2 The RURO model

In the present section, we present a version of the general RURO model applied to job choice as developed by Dagsvik and Strøm (1992, 2003, and 2006), Aaberge, Dagsvik and

[3] Statistics on Income and Living Conditions (SILC) is a survey held yearly in EU–member states and some other countries under supervision of EUROSTAT. Permission to use EU–SILC data for Belgium was obtained in the framework of the EUROMOD–project. More in formation on EU–SILC can befound on `ec.europa.eu/eurostat/web/microdata/european-union-statistics-on-income-and-living-conditions`. See Section 4 for more information on the data we used from EU-SILC.

[4] Currently males' educational attainment level is lagging behind that of females.

Strøm (1995), and Aaberge, Colombino and Strøm (1999). We first illustrate how the RURO model extends the choice problem of traditional labour supply models from a question of trading off leisure against consumption towards a model of job choice, against other non–market alternatives of time use (Subsection 2.1). Then, we discuss the preference part of the model (Subsection 2.2). Finally, we expose the modelling of opportunities (Subsection 2.3).

2.1 Opportunities and jobs

In general, the RURO model is an economic model of human choice behaviour. Human decision makers are assumed to choose the *best* element from a set of choice possibilities or opportunities, where 'best' is defined in terms of preferences (or, *vice versa*, preferences are derived from observed choice behaviour as that objective which would be maximised given those choices). Applied to job choice, the set of opportunities is to be thought of as a set of possible activities a particular individual might choose to execute. Some of these activities are rendered available through job offers. These job offers will be indexed by j, where the variable j belongs to an index set, $\mathcal{J}$ say. A job offer stipulates an amount of labour time to be supplied when accepting the offer, say h, and pays a wage, w. It is assumed that this wage can be expressed in units of time effectively spent on the job, so that (gross) revenues earned by the job equal the amount of time spent on the job times the wage.[5]

Gross earned labour income is then equal to wh. Gross earned labour income together with some other characteristics, say $\mathbf{x}_f$, among which non–earned gross income (exclusive of transfers), determine the outcome of the gross to net (disposable) income function $c = f(w, h; \mathbf{x}_f)$, where c stands for consumption which in a static model as the present one coincides with disposable income.[6] That is, saving is considered as part of consumption. The function f converts gross income components into net disposable income, by subtracting taxes to be paid and adding transfers and subsidies. Usually, the generation of disposable income is constructed from raw data on gross income, labour time and other characteristics, by means of a microsimulation model.

[5] This is generally not the case. Output dependent bonuses or piece–rates do not necessarily bear an obvious relation with time spent on the job. More surprisingly, in a regime with fixed monthly wages, the wage per unit of time is variable, since the number of hours a regular (that is: full time) job requires can differ over jobs, and for a specific job, the number of hours to be worked per month is not fixed. Finally, even if there is a fixed hourly wage, and no bonuses, there is no obvious way how to treat paid holidays. Should they be taken into account when calculating an hourly wage or not? When taking these into account, this would result in an increase of the gross hourly wage, as compared to the wage specified in the contract.

[6] We include the gross wage, and the number of hours worked as separate arguments in that function, as some aspects of the tax system, such as the Belgian work *bonus* may depend on the wage, rather than on labour income, wh. We are however aware that this might cause problems for the non–parametric identification of the RURO model.

Besides time spent on the job and the remuneration, jobs exhibit a number of other characteristics such as degree of responsibility, variation and challenge of the tasks, safety, healthiness, physical effort, stress, relation with colleagues and superiors. These characteristics will be denoted by $\mathbf{s}$. Preferences over these non–pecuniary attributes affect job choice.

One might also decline all job offers. Evidently, not executing a formal job does not require any time to be spent on the formal labour market ($h = 0$), and is assumed not to pay a wage ($w = 0$).A person who does not work, receives a net transfer (that is after deducting income taxes to be paid from her replacement income) equal to $f\left(0, 0; \mathbf{x}_f\right)$. In that case, time is spent on executing some of the available non–market opportunities. However, the set of activities[7] one has alternatively available is not the same for all individuals, neither is the extent to which a particular alternative is available. When living in a small town, attending concerts, theatre or visiting museums is certainly not as easy as for big city dwellers. If you are in a wheel chair, hiking is not an option. Which of the available non–market activities will be chosen, in case no job offer is accepted, again depends on preferences (or, *vice versa*, what one chooses to do allows to derive something on the shape of preferences that person supposedly has). Non–market alternatives will be indexed by i, belonging to the index set $\mathcal{I}$. The index set for jobs and non–market alternatives respectively, are disjoint: $\mathcal{I} \cap \mathcal{J} = \varnothing$. We will also use the index variable z to indicate an alternative in general, that is either a job or a non–market alternative. So, $z \in \mathcal{Z} := \mathcal{I} \cup \mathcal{J}$. To be really precise, an index z refers to a set of activities. If this set involves one or more jobs, the index z will belong to $\mathcal{I}$, while it belongs to $\mathcal{J}$ otherwise. An alternative including several part time jobs is described by the total number of hours these jobs involve and the hourly wage these pay together (calculated as earnings divided by total hours).

2.2 Random utility

In the model, preferences are defined over the number of hours h spent on jobs (which is zero if one chooses not to accept any job offer), consumption, c, and a set of other attributes, say $\mathbf{s}$, that a job or certain non–market activities possess, and that a person might care for. These other attributes are not observed by the researcher.

The observable, and thus from a behaviour theoretic point of view relevant, bundle of characteristics an alternative $z \in \mathcal{Z}$ exhibits, is denoted by $\left(C\left(z\right), H\left(z\right)\right)$, where $C\left(z\right)$ refers to the individually specific net disposable income resulting from executing activities indicated by z, and $H\left(z\right)$ to the labour time involved by the set of activities indicated by z. The utility

[7] The word 'activity' will be used here in a broad sense, including occupations which are not very 'active' such as sleeping and day dreaming. A certain type of agency or control is however presumed, since otherwise it would be difficult to talk about choice behaviour.

derived from these observable characteristics is denoted by $V(C(z), T - H(z); \mathbf{x}_V)$, where $\mathbf{x}_V$ are the specific values of a set preference shifters for the individual under consideration, and T denotes the number of time units available in the period over which labour time h is registered (*e.g.* 168 hours a week, if labour time is expressed in hours worked per week). It is assumed that the econometrician can derive some evidence on the shape of the function V on the basis of observations on $(c, T - h)$ and $\mathbf{x}_V$. So, no individual preference differences apart from those explained by observable characteristics $\mathbf{x}_V$, are allowed for in this part of the utility function, and V is therefore called the *systematic* part of the utility function, and, hence, of preferences.

Since the other attributes besides disposable income and labour time are not observable, their contribution to utility will be specified as a random term. Thus, when a set of activities z bears attributes $\mathbf{s} = s(z)$, the utility these attributes generate, is denoted by the random variable $\varepsilon(s(z))$. It is assumed that this utility from non–pecuniary attributes, $\varepsilon(s(z))$, enters overall utility of an alternative, z, in a multiplicatively separable way from the systematic part of the utility function. In order to make sense, this requires both, the systematic part of the utility, and the random term, to be non–negatively valued functions. In summary, the total utility derived from picking an alternative $z \in \mathcal{Z}$, denoted by $U(C(z), H(z), s(z); \mathbf{x}_V)$, equals:

$$U(C(z), H(z), s(z); \mathbf{x}_V) := V(C(z), T - H(z); \mathbf{x}_V) \cdot \varepsilon(s(z)). \tag{1}$$

As $c = f(w, h; \mathbf{x}_f)$, the systematic part of the utility function, $V(c, T - h; \mathbf{x}_V)$, induces a utility function, say Ψ, defined over hours worked on the formal labour market, h, and wage, w:

$$\Psi(w, h; \mathbf{x}_V, \mathbf{x}_f) := V(f(w, h; \mathbf{x}_f), T - h; \mathbf{x}_V). \tag{2}$$

Consequently, we can define preferences also in the space of hours of work, wage, and other attributes, as follows:

$$U(f(W(z), H(z); \mathbf{x}_f), H(z), s(z); \mathbf{x}_V) := \Psi(W(z), H(z); \mathbf{x}_V, \mathbf{x}_f) \cdot \varepsilon(s(z)), \tag{3}$$

where $W(z)$ is the wage paid by activity z. More in particular, for someone not accepting any job offer, and choosing an alternative $i \in \mathcal{I}$ which exhibits characteristics $s(i)$, the utility equals:

$$U(f(0, 0; \mathbf{x}_f), s(i); \mathbf{x}_V) = \Psi(0, 0; \mathbf{x}_V, \mathbf{x}_f) \cdot \varepsilon(s(i)). \tag{4}$$

The domain of the systematic part of the utility function in the wage–hours space, $\Psi(\cdot; \mathbf{x}_V, \mathbf{x}_f)$, is $[0, \infty) \times [0, T]$.

From now on we will drop the conditioning variables $\mathbf{x}_V$ and $\mathbf{x}_f$ in the part of the utility functions.

2.3 Random opportunities

Both, jobs and non–market activities, are not equally available to all individuals. This is captured by the notion of intensity with which alternatives are rendered available to a specific individual. The probability to receive a job offer as a civil engineer, for someone who has only completed secondary school, is *e.g.* zero. Something similar holds for non–market activities: they are not all equally available to all agents. Someone having lost her legs will not be able to run (or not in the same fashion as before), though she might continue to be fond of it.[8]

The intensity with which a job is offered to an individual depends on a number of personal characteristics, such as skills, education, experience, and on the characteristics of the job itself, more specifically, the wage, the labour time regime of the job, and its other attributes. For the sake of simplifying notation we will drop these conditioning variables.

In equation (3), preferences were defined over the *continuous* set of all possible amounts of time spent on the jobs. In the real world, however, jobs requiring a non–rational number of hours a week, are not available. Be it alone to organise the production process, it might sometimes be required to have a number of people working together during a fixed number of hours. So, in practice, full–time, three–quarter time, half time, one–quarter, or 20% jobs are more densely offered than other labour time regimes. Let $g_2(h)$ be the density with which jobs requiring h hours of labour supply, are rendered available to a specific individual. Similarly, jobs pay different wages. The conditional density with which, among the job offers requiring h hours of work, those that are paying a wage equal to w, are offered to a specific individual, is denoted by $g_1(w\,|h)$.

Persons do not only care for the wages a job pays, and the number of hours to be worked, but also for the other attributes of a job. From a behavioural theoretic point of view (see equation 1), these are only important in as far as they yield a specific value for the multiplication factor in the utility function for those alternatives. Two jobs, j_1 and j_2 say, paying the same wage and requiring the same amount of hours, with attributes yielding the same value of the multiplication factor in the utility function for those alternatives, that is $\varepsilon(s(j_1)) = \varepsilon(s(j_2))$, are thus, according to the behavioural model of equation (1), equivalent to each other in the present model, and therefore will be considered as the same opportunity. The variable indicating the value of this multiplication factor is denoted as υ.

The intensity with which job offers yielding a value υ, are arriving to a specific individual is

[8] In the supplementary material *on line* Appendix I we provide a brief introduction to the type of stochastic process that describes the degree to which job offers and non–market alternatives become available to an individual, and which is known as an inhomogeneous spatial Poisson process.

denoted by $\lambda^1(\upsilon)$.[9] We assume the following functional form for λ^1:

$$\lambda^1(\upsilon) = \frac{\pi_1}{\upsilon^2}, \tag{5}$$

where π_1 is a probability measure for the proportion of capacities and characteristics that are useful on the job market, an individual is endowed with.

The functional form of λ^1 implies that attributes which are particularly disliked (yield a zero or very small value for υ) are excessively abundant, while those that are particularly liked (yielding a very high value of υ), are extremely scarce. This functional form guarantees independence of irrelevant alternatives in the probability of choosing jobs (Dagsvik, 1994). The distinguishing value of different potential non–market activities, is completely absorbed by the different values of the multiplication factor in the utility function they generate. Indeed, the systematic part of the utility function is for all non–market alternatives equal to $\Psi(0,0)$. As for jobs with the same wage and required labour input, two non–market activities, i_1 and i_2 say, with attributes yielding the same value of the multiplication factor in the utility function for those alternatives, that is $\varepsilon(s(i_1)) = \varepsilon(s(i_2))$, are from a behavioural theoretic point of view equivalent to each other, and will therefore be considered as one and the same opportunity. For the same reason as in the case of job offers, it will be assumed that leisure activities which are particularly disliked, are abundantly available, while those that are intensely desired, are rather difficult to obtain. As π_1 is a probability measure for the proportion of capacities and characteristics that are useful on the job market, π_0, which is the proportion of an individual's capacities and characteristics that are currently not useful on the job market, can be defined as: $\pi_0 := 1 - \pi_1$. The intensity with which non–market activities yielding a multiplication factor equal to ϵ, denoted by $\lambda^0(\epsilon)$, are accessible to an individual, is thus assumed to be equal to:

$$\lambda^0(\epsilon) = \frac{\pi_0}{\epsilon^2}. \tag{6}$$

Let $\mathcal{E}_u$ bet the set of values for ϵ such that non–market activities yield at least a utility level larger than or equal to u: $\mathcal{E}_u = \{\epsilon \in \mathbb{R}_+ \,|\Psi(0,0)\,\epsilon \geq u\}$. Define $\Lambda^0(\mathcal{E}_u)$, as:

$$\Lambda^0(\mathcal{E}_u) \;:=\; \int_{u/\Psi(0,0)}^{\infty} \tfrac{\pi_0}{\epsilon^2}\,\mathrm{d}\,\epsilon \;=\; \pi_0 \tfrac{\Psi(0,0)}{u}. \tag{7}$$

If one assumes that λ^0 is the intensity measure of an inhomogeneous spatial Poisson process, then the number of non–market alternatives that yield a utility level of at least u, available

[9] Job offer arrivals depend on personal capacities and skills which are subdivided in those apt to execute formal jobs, and those more suited for performing leisure activities. Next there may be personal characteristics on the basis of which discrimination in job offers by employers might take place. Again, we omit the impact of these conditioning variables for the sake of notational simplicity.

to a person is Poisson distributed (see the *on line* supplementary material Appendix I). Let the number of available non–market activities yielding a utility level of at least u, be denoted by $N(\Psi(0,0)\epsilon \geq u)$. The probability that $N(\Psi(0,0)\epsilon \geq u) = n$, is, according to the Poisson distribution, equal to:

$$P(N(\Psi(0,0)\epsilon \geq u) = n) = \frac{\left(\Lambda^0(\mathcal{E}_u)\right)^n \exp\left[-\Lambda^0(\mathcal{E}_u)\right]}{n!} = \frac{(\pi_0\Psi(0,0)/u)^n \exp[-\pi_0\Psi(0,0)/u]}{n!}. \tag{8}$$

The value of $\Lambda^0(\mathcal{E}_u)$ equals the expected number of non-market opportunities available to an individual, yielding a utility level at least equal to u. The higher the value of $\Lambda^0(\mathcal{E}_u)$, the more skewed to the right this distribution becomes, that is, the higher the probability that the number of available non–market alternatives yield a utility level of at least u is relatively big.

The probability that all available non–market alternatives to an individual, yield a utility level lower than u, equals the probability that the number of available alternatives with utility larger than or equal to u, is zero:

$$P(\Psi(0,0)\epsilon < u) = P(N(\Psi(0,0)\epsilon \geq u) = 0) = \exp\left[-\Lambda^0(\mathcal{E}_u)\right] = \exp\left[-\frac{\pi_0\Psi(0,0)}{u}\right]. \tag{9}$$

From the last equation, it can be concluded that the utility that can be derived from the available non–market alternatives, which is a stochastic variable denoted by $U_0 = \Psi(0,0)\epsilon$, is Fréchet distributed with location parameter $\mu = 0$, scale parameter $\sigma_0 = \pi_0\Psi(0,0)$, and shape parameter $\alpha = 1$.[10] This will prove useful when formulating the likelihood function in the next section (Section 3.1).

Let $\mathcal{H}$ be the set of all possible labour time regimes of jobs offered in the market, and $\mathcal{W}$ the set of possible wage offers. Wages can obtain any positive value. Let $\mathcal{B} := \mathcal{B}_h \times \mathcal{B}_w$ be the Cartesian product of a measurable subset of labour time regimes $\mathcal{B}_h \subseteq \mathcal{H}$, and wage offers $\mathcal{B}_w = (0, w)$, for some positive w. Analogously to the modelling of the availability of non–market opportunities, the arrival of job offers[11] to an individual, is modelled by an inhomogeneous spatial Poisson process. Events are job offers that are characterised by a labour time regime h, a wage offer w, and the utility that can be derived from other attributes υ. The intensity parameter of this Poisson process is equal to $g_2(h)\, g_1(w\,|h)\, \lambda^1(\upsilon)$. Define next the set of job offers specifying labour time regime $t \in \mathcal{B}_h$, paying a wage r lower than w (that is $r \in \mathcal{B}_w$), and which will yield a utility level at least equal to u, as

[10] In general, the class of Fréchet distributions is defined as: $F(x;\mu,\sigma,\alpha) := \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right]$, where μ is a location parameter, σ a scale parameter, and α is a shape parameter. We could alternatively argue that the multiplier in the utility function, stemming from the attractiveness of the non–pecuniary attributes of non–market alternatives to a particular individual is Fréchet distributed with location parameter $\mu = 0$, scale parameter $\sigma = \pi_0$, and shape parameter $\alpha = 1$.

[11] As mentioned before, a 'job offer' is a short hand for 'an alternative containing at least one job offer'.

$\mathcal{U}_{h,w,u} := \{(t,r,\upsilon) \in \mathcal{B}_h \times \mathcal{B}_w \times \mathbb{R}_+ |\, \Psi(r,t)\,\upsilon \geq u\}$, and define:

$$\Lambda^1(\mathcal{U}_{h,w,u}) := \int\limits_{t\in\mathcal{B}_h} g_2(t) \int\limits_{r\in\mathcal{B}_w} g_1(r\,|t) \int\limits_{u/\Psi(r,t)}^{\infty} \tfrac{\pi_1}{\upsilon^2}\, \mathrm{d}\upsilon\, \mathrm{d}r\, \mathrm{d}t \tag{10}$$

$$= \frac{\int_{t\in\mathcal{B}_h} g_2(t) \int_{r\in\mathcal{B}_w} g_1(r|t)\pi_1\Psi(r,t)\,\mathrm{d}r\,\mathrm{d}t}{u}.$$

Let $N(\mathcal{B},u)$ be the number of job offers with a wage r belonging to $\mathcal{B}_w$, the number of hours to be worked in $\mathcal{B}_h$, and yielding a utility level larger than or equal to u. The probability for an individual to be offered n such jobs is under the present assumptions governed by Poisson distribution, and is thus equal to:

$$P(N(\mathcal{B},u)=n) = \frac{\left(\Lambda^1(\mathcal{U}_{h,w,u})\right)^n \exp\left[-\Lambda^1(\mathcal{U}_{h,w,u})\right]}{n!}. \tag{11}$$

Again, $\Lambda^1(\mathcal{U}_{h,w,u})$ can be interpreted as the expected number of job offers with labour time regime, wage, and utility level in $\mathcal{U}_{h,w,u}$.

Let $P(U(r,t) < u\,|(t,r) \in \mathcal{B})$ be the probability that all job offers with wages in $\mathcal{B}_w$, and number of hours to be worked in $\mathcal{B}_h$, yield a utility level *less than* u. This is equal to the probability that there are no job offers available that pay wages and specifying a working time in these ranges, and which yield a utility level of at least u:

$$P(U(r,t) < u\,|(t,r)\in\mathcal{B}) = P(N(\mathcal{B},u)=0) = \exp\left[-\Lambda^1(\mathcal{U}_{h,w,u})\right]$$

$$= \exp\left[-\frac{\int_{t\in\mathcal{B}_h} g_2(t) \int_{r\in\mathcal{B}_w} g_1(r\,|t)\pi_1\Psi(r,t)\,\mathrm{d}r\,\mathrm{d}t}{u}\right]. \tag{12}$$

The utility level that can be obtained from jobs with working time and wage combinations in $\mathcal{B}$, which is a stochastic variable, denoted as $U_{\mathcal{B}}$, and which is governed by the stochastic process of job offers arriving to an individual, is thus Fréchet distributed with location parameter $\mu = 0$, scale parameter

$$\sigma_{\mathcal{B}} = \int\limits_{t\in\mathcal{B}_h} g_2(t) \int\limits_{r\in\mathcal{B}_w} g_1(r\,|t)\,\pi_1\Psi(r,t)\;\mathrm{d}\,r\,\mathrm{d}\,t,$$

and shape parameter $\alpha = 1$.

The derivation of this distribution is equally valid for any (measurable) subset $\mathcal{B}$ of the space of possible working times and wage combinations job offers might exhibit. More in particular, it holds for the complement of $\mathcal{B}$ in the set of all possible working time wage combinations, defined as $\mathcal{B}^c := \mathcal{B}_h^c \times \mathcal{B}_w^c = \{\mathcal{H} \setminus \mathcal{B}_h\} \times [w, \infty)$. It follows that the random variable $U_{\mathcal{B}^c}$, denoting the utility level derivable from possible job offers with working time wage combinations in $\mathcal{B}^c$, is Fréchet distributed with location parameter $\mu = 0$, scale parameter

$$\sigma_{\mathcal{B}^c} = \int\limits_{t \in \mathcal{B}_h^c} g_2\left(t\right) \int\limits_{r \in \mathcal{B}_w^c} g_1\left(r\,|t\,\right) \pi_1 \Psi\left(r, t\right)\, \mathrm{d}\, r\, \mathrm{d}\, t,$$

and shape parameter $\alpha = 1$.

3 Likelihood function, functional form, and estimation

3.1 Likelihood function

Now we turn to the behavioural implications of the model explained in Section 2. From the available job offers and non–market opportunities, a person will choose that alternative she likes most. The probability that this will be an alternative including a job offer with a working time wage combination in the set $\mathcal{B}$, is equal to the probability that $U_{\mathcal{B}} \geq \max\{U_0, U_{\mathcal{B}^c}\}$. As the processes governing the arrival of job offers and non–market opportunities are assumed to be independent, the probability that $U_{\mathcal{B}}$ (the utility from a job offer with a labour time and wage combination in $\mathcal{B}$) is equal to or greater than $\max\{U_0, U_{\mathcal{B}^c}\}$ is equal to the product of the probability that $U_{\mathcal{B}}$ is greater than or equal to U_0 and the probability that $U_{\mathcal{B}}$ is greater than or equal to $U_{\mathcal{B}^c}$. That amounts to:

$$\begin{aligned} P\left(U_{\mathcal{B}} \geq \max\{U_0, U_{\mathcal{B}^c}\}\right) &= \int\limits_0^\infty \tfrac{\sigma_{\mathcal{B}}}{(u)^2} \exp\left[-\tfrac{\sigma_{\mathcal{B}}}{u}\right] \exp\left[-\tfrac{\sigma_{\mathcal{B}^c}}{u}\right] \exp\left[-\tfrac{\sigma_0}{u}\right] \mathrm{d}\, u \\ &= \int\limits_0^\infty \tfrac{\sigma_{\mathcal{B}}}{(u)^2} \exp\left[-\left(\tfrac{\sigma_{\mathcal{B}}+\sigma_{\mathcal{B}^c}+\sigma_0}{u}\right)\right] \mathrm{d}\, u \\ &= \tfrac{\sigma_{\mathcal{B}}}{\sigma_{\mathcal{B}}+\sigma_{\mathcal{B}^c}+\sigma_0} \\ &= \frac{\int_{t \in \mathcal{B}_h} g_2(t) \int_{r \in \mathcal{B}_w} g_1\left(r\,|t\,\right) \pi_1 \Psi\left(r, t\right)\, \mathrm{d}\, r\, \mathrm{d}\, t}{\pi_0 \Psi(0,0) + \int_{t \in \mathcal{H}} g_2(t) \int_{r \in \mathcal{W}} g_1(r|t\,) \pi_1 \Psi(r,t)\, \mathrm{d}\, r\, \mathrm{d}\, t}. \end{aligned} \tag{13}$$

In a similar fashion it can be derived that the probability to choose a non–market alternative, is equal to the probability that U_0 is equal to or greater than $U_{\mathcal{B}\cup\mathcal{B}^c}$, which is equal to:

$$P\left(U_0 \geq U_{\mathcal{B}\cup\mathcal{B}^c}\right) \quad = \quad \frac{\pi_0 \Psi(0,0)}{\pi_0 \Psi(0,0) + \int_{t\in\mathcal{H}} g_2(t) \int_{r\in\mathcal{W}} g_1(r|t)\pi_1 \Psi(r,t)\,\mathrm{d}\,r\,\mathrm{d}\,t}. \tag{14}$$

We indicated before that the endowment of capacities and characteristics useful on the job market is defined relatively to total capacities and characteristics, valuable either for market or non-market activities. Equations (13) and (14) make clear that this is necessary for identifying the model. Indeed, the same result would be obtained when dividing through the numerator and denominator by π_0. We therefore introduce the notion of *relative* intensity with which job offers arrive, as compared to the degree to which non–market alternatives are available:

$$q := \frac{\pi_1}{\pi_0}. \tag{15}$$

As it was assumed that $0 \leq \pi_i \leq 1$ for $i = 0, 1$, and that $\pi_1 + \pi_0 = 1$, both π_1 and π_0 can be recovered from q.
An additional assumption for identification is the independence of the wage offer distribution from the hours specified by the job offers. That is $g_1\left(w\,|h\right) = g_1\left(w\right)$, $\forall\, h \in \mathcal{H}$.
The likelihood[12] that an individual will choose one particular job offer requiring labour time h, and paying a wage w, can thus be obtained from equation (13):

$$\varphi\left(w, h\right) = \frac{\Psi\left(w, h\right)\; q\, g_1\left(w\right) g_2\left(h\right)}{\Psi\left(0, 0\right) + \int\limits_{r\in\mathcal{W}} \int\limits_{t\in\mathcal{H}} \Psi\left(r, t\right)\; q\, g_1\left(r\right) g_2\left(t\right)\;\mathrm{d}\,t\;\mathrm{d}\,r}, \tag{16}$$

Similarly, the likelihood and individual's most preferred non–market alternative is preferred to any of the job offers, equals:

$$\varphi\left(0, 0\right) = \frac{\Psi\left(0, 0\right)}{\Psi\left(0, 0\right) + \int\limits_{r\in\mathcal{W}} \int\limits_{t\in\mathcal{H}} \Psi\left(r, t\right)\; q\, g_1\left(r\right) g_2\left(t\right)\;\mathrm{d}\,t\;\mathrm{d}\,r}. \tag{16'}$$

We exclusively concentrated on individual decision makers. The model is easily extended to the case of households consisting of couples (with or without children), if one is willing to assume a unitary decision making model. We provide this extension in the *on line* supplementary Appendix II.

[12] We use the term likelihood, though, in econometrics, the likelihood would express this as function of the parameters of the model to be estimated.

It is worthwhile to compare the likelihood function (16)–(16) with what is obtained in a random utility function based upon discrete choice of labour time regimes (such as in Van Soest, 1995). In this approach, the wage a person obtains, is, apart from measurement problems (see footnote 2), a fixed individual characteristic reflecting that person's productivity. Choice of labour time is free but limited to a discrete set $\{h_k; k = 1, 2, \ldots, K\}$. Under the assumption that the stochasic parts of the utility functions are Fréchet distributed, the likelihood (probability) to observe an individual choosing a labour time regime h_l equals:

$$\varphi(w, h_l) = \frac{\Psi(w, h_l)}{\psi(0,0) + \sum_{k=1}^{K} \Psi(w, h_k)}. \tag{17}$$

The difference with (16)–(16) is twofold. In the RURO model utilities are weight with the intensity with which alternatives are rendered available to an individual. Next, the wage is part of the job offer. Consequently the denominator sums over all possible pairs of wages and labour time regimes, (w, h) job offers contain, and not only over possible labour time regimes for a given wage.

3.2 Identification

Some parts of the model are non–parametrically identified. A fuller treatment of this issue is provided in Aaberge, Columbino and Strøm (1999), the working paper version of Dagsvik and Strøm (2006) (See Dagsvik and Strøm, 2004), and Dagsvik and Jia (2014). The main line of argument for identifying the wage offer distribution from preferences runs as follows. Isolate two observationally equivalent groups of individuals in the population, supplying different number of hours, say h_1 and h_2, for the same wage w. The relative proportion of these groups in the population is $\varphi(w, h_1) / \varphi(w, h_2)$, which according to the model in equation (16) reduces to:

$$\frac{\varphi(w, h_1)}{\varphi(w, h_2)} = \frac{\Psi(w, h_1)\, g_2(h_1)}{\Psi(w, h_2)\, g_2(h_2)}. \tag{18}$$

Doing this for different levels of wages, allows to identify the function $\Psi(w, h)\, g_2(h)$. Looking then at persons performing the same number of hours, but accepting different wages, w_1 and w_2 say, gives:

$$\frac{\varphi(w_1, h)}{\varphi(w_2, h)} = \frac{\Psi(w_1, h)\, g_2(h)}{\Psi(w_2, h)\, g_2(h)} \frac{g_1(w_1)}{g_1(w_2)}. \tag{19}$$

As $\Psi(w, h)\, g_2(h)$ was already identified in the previous step, it is now possible to identify $g_1(w)$, using the fact that it is a density, and thus that $\int_{w \in \mathcal{W}} g_1(w)\, \mathrm{d}\, w = 1$.

Then, consider an observationally equivalent group of persons in the population. Some of them will be engaged in a formal job, and some of them not. The relative proportion of those groups in the population are:

$$\frac{\varphi(w,h)}{\varphi(0,0)} = \frac{\Psi(w,h)\ q\, g_1(w)\, g_2(h)}{\Psi(0,0)}. \tag{20}$$

As Ψ is a utility function, we can normalize the value of $\Psi(0,0)$, which allows to identify q from this equation. In our empirical application, we tried to improve upon the non-parametric identification of q by introducing an exclusion restriction. More specifically, a group specific unemployment rate[13] is added as an explanatory variable for q. We assume that this variable does not affect individual preferences, but, obviously, it has some relation with the structure of labour demand.

The utility function $\Psi(w,h)$ and the distribution of offered labour time regimes $g_2(h)$ are however not separately non–parametrically identified. One way out is to give a more fundamental justification of the functional preferences used. For example, Dagsvik and Røine Hoff (2011) and Dagsvik (2013) give a non–parametric justification of the preferences embodied by the Box–Cox type of utility functions that we will use (see the next section).

Moreover, it can be argued that the occurrence of peaks in the distribution of the number of hours worked, as observed in many datasets, around half time, three quarter time and full time work, are not easily explained by the traditional way preferences are shaped in economics, neither by the kinks in the shape of the budget set caused by different tax structures.

3.3 Functional forms

In the present section we present the functional forms of the different components of the model that will be used in the empirical application in Section 5.

At the preference side,

- the systematic part of the log utility function for singles is of the Box–Cox type: $\ln V(c, T-h; \mathbf{x}_V) = \beta_c \cdot \left(\frac{c^{\alpha_c}-1}{\alpha_c}\right) + (\boldsymbol{\beta}_h' \mathbf{x}_V) \cdot \left(\frac{((T-h)/T)^{\alpha_h}-1}{\alpha_h}\right)$, with $\alpha_c, \alpha_h < 1$. Intensity of preferences for leisure is increasing (decreasing) in an element of $\mathbf{x}_V$, if the associated parameter of $\boldsymbol{\beta}_h$ is positive (negative).[14] The exponents, α_c and α_h, determine the curvature of the indifference curves in terms of labour time and consumption (that is, while keeping other attributes affecting preferences fixed). The lower these are, the less substitutable leisure and consumption are;

[13] Ideally one would use the number of vacancies for suitable jobs for certain identifiable groups of persons in the population, but, such information is not really observable.

[14] More details are provided in Section 5.1.

– for couples, a unitary decision model is assumed, but spouses' leisure time is considered to be an assignable good. So, the systematic part of preferences is defined over consumption and each spouse's leisure time. Partner's time endowments are equal. An interaction term capturing potential complementarities between partners' leisure time is added to the utility function:

$$\ln V_g\left(c, T-h_1, T-h_2; \mathbf{x}_V\right) = \beta_{c,g} \cdot \left(\frac{c^{\alpha_{c,g}}-1}{\alpha_{c,g}}\right) + \sum_{i=1,2} \left(\boldsymbol{\beta}'_{h_i}\mathbf{x}_V\right) \cdot \left(\frac{\left((T-h_i)/T\right)^{\alpha_{h_i}}-1}{\alpha_{h_i}}\right) + \beta_{h_1,h_2} \cdot \prod_{i=1,2} \left(\frac{\left((T-h_i)/T\right)^{\alpha_{h_i}}-1}{\alpha_{h_i}}\right),$$

with $\alpha_{c,g}, \alpha_{h_i} < 1$ $(i = 1, 2)$. The interpretation of the exponents and the $\boldsymbol{\beta}'_{h_i}$ $(i = 1, 2)$ remains the same as for singles; in addition, $\beta_{h_1,h_2} > (<) 0$ means that partners' leisure are complements (substitutes).

At the opportunity side,

– the log of the intensity of job offers relative to the availability of non–market alternatives is linear in the covariates: $\ln q\left(\mathbf{x}_{opp}\right) = \boldsymbol{\eta}'_q \mathbf{x}_{opp}$. The vector $\mathbf{x}_{opp}$ resumes covariates that might affect the job offer intensity, and should contain a constant term, and the associated coefficient is denoted by $\eta_{q,0}$;

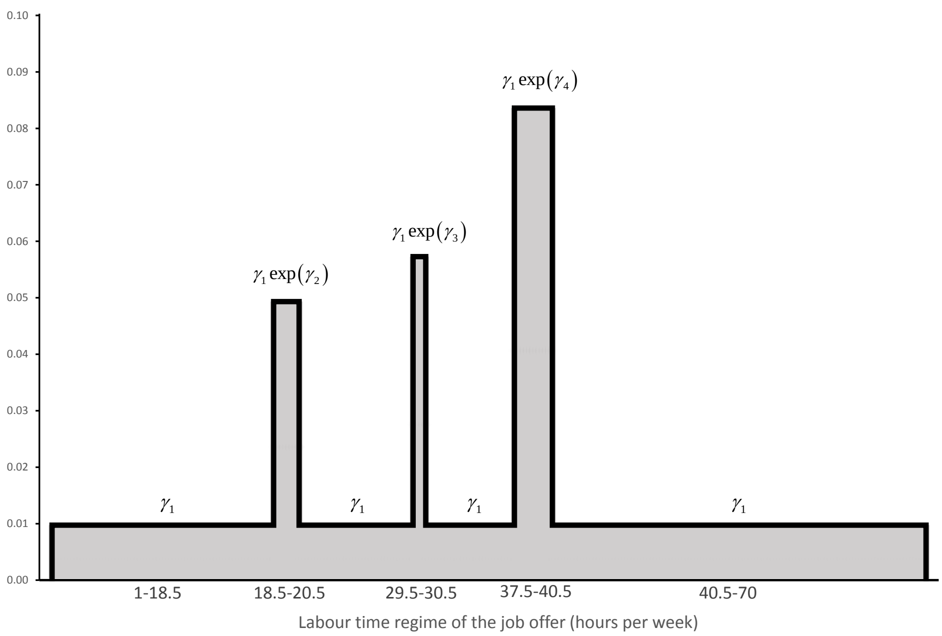
– the wage density $g_1\left(w; \mathbf{x}_w\right)$ is assumed to be lognormal:
$g_1\left(w; \mathbf{x}_w\right) = \frac{1}{w \cdot \sigma \cdot \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln w - \boldsymbol{\delta}'_{g_1}\mathbf{x}_w}{\sigma}\right)^2\right)$; and is dependent on covariates $\mathbf{x}_w$ that might affect the wage offer distribution;

– the distribution of the labour time regimes offered, is piecemeal uniform. There are a number of, say K, peaks, indexed by $k = 1, 2, \ldots, K$, around which the bulk of the job offers' labour time regimes are concentrated (typically around half time, that is 18.5 to 20.5 hour a week in our application, three quarter time, or 29.5 to 30.5 hours a week, and full time, or 37.5 to 40.5 hours). The lower and upper bound of peak k $(k = 1, 2, \ldots, K)$ are denoted by respectively $\underline{H}_k$ and $\overline{H}_k$. There is a lower limit, $H_{\min}$, below which job offers are not considered to belong to the formal labour market (fixed at one hour a week in the application below); and an upper limit of labour time spent on formal jobs, denoted by $H_{\max}$, and fixed at 70 hours per week in our application. This results in the following density function:

$$g_2\left(h; \mathbf{x}_h\right) = \begin{cases} \gamma_1 & \text{if } \; h \in \left[H_{\min}, \underline{H}_1\right[, h \in \left[\overline{H}_k, \underline{H}_{k+1}\right[, \text{ or } \; h \in \left[\overline{H}_K, H_{\max}\right[, \\ & \qquad k = 1, 2, \ldots, K-1, \\ \gamma_1 \exp \gamma_{k+1} & \text{if } \; h \in \left[\underline{H}_k, \overline{H}_k\right[, \quad k = 1, 2, \ldots, K. \end{cases}$$

The only covariate in $\mathbf{x}_h$ allowed to affect this function is the sex of the person in our specification. An example of such a distribution function is given in Figure 1.

Figure 1: Peak distribution for labour time regimes



3.4 Estimation

To estimate the parameters governing preferences, the relative intensity of market over non–market alternatives, and the distribution of wage offers and labour time regimes, a likelihood function, say $\mathcal{L}$, is constructed on the basis of equations (16), (16') (for the case of the couples, see (16") of the *on line* supplementary material Appendix II). The individual contributions of a single to that likelihood function are indeed composed of the likelihood that the observed choice is the most preferred one, reflected in equations (16), or (16'), depending on whether the observed choice involves participation on the formal labour market executing a job (or set of jobs) requiring h hours of work, and paying a wage w, or whether it is the non–market alternative. In these expressions, the numerator is thus evaluated at the actually observed choice, when constructing the likelihood function. Similarly, for couples the first, second, third, or fourth equation in (16") (in the *on line* supplementary material Appendix II) applies, dependent on whether both partners, only partner j ($j = 1, 2$), or none of both actually are engaged in formal jobs.

In practice, we do not observe the set of wage offers, $\mathcal{W}$, nor the offered labour time regimes, $\mathcal{H}$. Therefore, a set of alternatives in the space of wages and labour time regimes is sampled from a prior density function, say $\mathbb{P}(w,h)$. Denote the set of sampled combinations of wage offers and labour time regimes, possibly including the non–market alternative, by $\mathcal{D}$. The observed choice $\left(w^{\text{obs}}, h^{\text{obs}}\right)$ is to be always included in the sampled choice set. From the sampling densities $\mathbb{P}(w,h;\mathbf{x}_{\mathbb{P}})$, the likelihood to sample a set $\mathcal{D}$ given that the observed choice equals $\left(w^{\text{obs}}, h^{\text{obs}}\right)$ can be constructed. [15] It is denoted by $\mathcal{P}\left(\mathcal{D}\,\middle|\left(w^{\text{obs}}, h^{\text{obs}}\right)\right)$, and it equals:

$$\mathcal{P}\left(\mathcal{D}\,\middle|\left(w^{\text{obs}}, h^{\text{obs}}\right)\right) := \prod_{i:(w_i,h_i)\in\mathcal{D}} \frac{\mathbb{P}(w_i,h_i)}{\mathbb{P}\left(w^{\text{obs}}, h^{\text{obs}}\right)}. \tag{21}$$

Recall that the probability (density) that a job paying a wage w, and requiring a number of h hours to be worked, would be optimal given a choice set $\mathcal{C} := \{0,0\} \cup \mathcal{W} \times \mathcal{H}$, was derived in equations (16), and (16') if the non–market alternative would be the most preferred option. The unconditional probability to sample a choice set $\mathcal{D}$, denoted by $\Pi(\mathcal{D})$, can thus be written as:

$$\Pi(\mathcal{D}) = \sum_{i:(w_i,h_i)\in\mathcal{D}} \mathcal{P}\left(\mathcal{D}\,|(w_i,h_i)\right)\varphi(w_i,h_i)\,. \tag{22}$$

Using Bayes' law, the probability (density) to observe an agent choosing a job offer that pays a wage w_i and requires h_i hours of labour time from the sampled set $\mathcal{D}$, thus equals:

$$\widetilde{\varphi}(w_i,h_i\,|\mathcal{D}) = \frac{\mathcal{P}\left(\mathcal{D}\,|(w_i,h_i)\right)\varphi(w_i,h_i)}{\Pi(\mathcal{D})}. \tag{23}$$

Using equations (21) and (22), we can thus reformulate the simulated likelihood to observe someone choosing an alternative (w,h) from a choice set $\mathcal{D}$ sampled according to the prior $\mathbb{P}(w,h)$, as:

$$\begin{aligned}\widetilde{\varphi}(w,h\,|\mathcal{D}) &= \frac{\Psi(w,h)/\mathbb{P}(w,h)}{\Psi(0,0)/\mathbb{P}(0,0) + \sum_{(r,t)\in\mathcal{D}\setminus\{(0,0)\}} \Psi(r,t)\;q\;g_1(r)g_2(t)/\mathbb{P}(r,t)} \\ &= \frac{\Psi(w,h)\,q\;g_1(w)g_2(h)\frac{\mathbb{P}(0,0)}{\mathbb{P}(w,h)}}{\Psi(0,0) + \sum_{(r,t)\in\mathcal{D}\setminus\{(0,0)\}} \Psi(r,t)\;q\;g_1(r)g_2(t)\frac{\mathbb{P}(0,0)}{\mathbb{P}(r,t)}}.\end{aligned} \tag{24}$$

The corresponding expression for choosing the non–market alternative equals:

$$\widetilde{\varphi}(0,0\,|\mathcal{D}) = \frac{\Psi(0,0)}{\Psi(0,0) + \sum_{(r,t)\in\mathcal{D}\setminus\{(0,0)\}} \Psi(r,t)\;q\;g_1(r)g_2(t)\frac{\mathbb{P}(0,0)}{\mathbb{P}(r,t)}}. \tag{24'}$$

[15] The issue of sampling choice sets for estimating the RURO model is discussed more in detail in McFadden (1978), Ben–Akiva and Lerman (1985), Aaberge, Columbino and Wennemo (2009), Train (2009), and Lemp and Kockelman (2012).

One further normalisation issue is in order. Note that the constant term of the $q(\mathbf{x}_{opp})$–function, $\exp(\eta_{q,0})$, occurs in any term of the likelihood where γ_1 appears (that is, in those terms of the sum pertaining to a job offer on the formal labour market, $(w,h) : w,h > 0$), and each time these terms appear as a product. Therefore both, $\eta_{q,0}$ and γ_1, cannot be estimated separately. But γ_1 is still identified by the definition:

$$\gamma_1 \left(H_{\max} - \overline{H}_K + \sum_{k=1}^{K-1} \left(\underline{H}_{k+1} - \overline{H}_k \right) + \underline{H}_1 - H_{\min} + \sum_{k=1}^{K} \left(\overline{H}_k - \underline{H}_k \right) \exp \gamma_k \right) \equiv 1. \tag{25}$$

This means in practice that one does not estimate all the parameters in the likelihood (24) (respectively 24'), but rather reduces these equations to:

$$\widetilde{\varphi}(w,h\,|\mathcal{D}) = \frac{\Psi(w,h)\ \widetilde{q}\ g_1(w) \mathscr{g}_2(h) \frac{\mathbb{P}(0,0)}{\mathbb{P}(w,h)}}{\Psi(0,0) + \sum\limits_{(r,t)\in\mathcal{D}\setminus\{(0,0)\}} \Psi(r,t)\ \widetilde{q}\ g_1(r) \mathscr{g}_2(t) \frac{\mathbb{P}(0,0)}{\mathbb{P}(r,t)}}, \tag{26}$$

where:

$$\begin{aligned} \mathscr{g}_2(h) &= \frac{g_2(h)}{\gamma_1}, \\ \widetilde{q} &= \gamma_1 q. \end{aligned}$$

For the likelihood to choose a non–market alternative, this becomes:

$$\widetilde{\varphi}(0,0\,|\mathcal{D}) = \frac{\Psi(0,0)}{\Psi(0,0) + \sum\limits_{(r,t)\in\mathcal{D}\setminus\{(0,0)\}} \Psi(r,t)\ \widetilde{q}\ g_1(r) \mathscr{g}_2(t) \frac{\mathbb{P}(0,0)}{\mathbb{P}(r,t)}}, \tag{26'}$$

One is able to back out an estimate of $\eta_{q,0}$ by using equation (25). Indeed, subtracting $\ln \widehat{\gamma}_1$ (with $\widehat{\gamma}_1$, the estimated value of γ_1 from applying equation (25) using the estimates for γ_{k+1}, for $k = 1, 2, \ldots, K$) from the estimated constant of $\ln(\widetilde{q}(\mathbf{x}_{opp}))$.

We used the following specification of $\mathbb{P}(w,h)$ for constructing a choice set $\mathcal{D}$: wages are sampled from a lognormal distribution with parameters μ and ς, labour time is sampled from the uniform distribution on the $[H_{\min}, H_{\max})$–interval, and the probability to sample non–market alternatives is the observed inactivity degree in the sample (that is the relative number of persons in the sample being engaged in formal jobs for less than one hour a week), say π_0^{obs}.[16] That is:

$$\begin{aligned} \mathbb{P}(w,h) &= \pi_0^{\text{obs}} && \text{if } (w,h) = (0,0), \\ &= \left(1 - \pi_0^{\text{obs}}\right) \frac{\left(w\varsigma\sqrt{2\pi}\right)^{-1} \exp\left(-\frac{(\ln w - \mu)^2}{2\varsigma^2}\right)}{H_{\max} - H_{\min}} && \text{if } w > 0, h \in [H_{\min}, H_{\max}[, \\ &= 0 && \text{otherwise.} \end{aligned} \tag{27}$$

[16] The exact figures are $\pi_0^{\text{obs}} = .104$, $\mu = 2.71$, and $\varsigma = .308$ for males, and the corresponding number for females are .246, 2.63, and .297.

3.5 Simulation

In order to evaluate the fit of the estimates, or the estimated model's prediction of behavioural reactions to changes in explanatory variables, a simulation method can be used. Thereto, a choice set is to be drawn (possibly capturing changes in the intensity with which certain alternatives become available to certain persons) and then it is determined what an agent's best choice would be within this simulated choice set, according to the estimated preferences of that person. If simulation is used for evaluating the fit of the estimated model, then the choice set is drawn according to the model estimates (the relative intensity of job offers, the wage offer distribution, and the labour time regime offers), and the simulated choices from that set are to be compared with actual ones, as observed in the data. This is done in Figures 6–11 below (Section 6.1). Next we will use the simulation method also for calculating elasticities, and to evaluate the effects of changes in the education level on labour market participation (see Sections 6.2 and 7).

When simulating to assess the fit, one uses the estimated measure of intensity with which alternatives (w, h) are offered to an agent, that is, using the estimates of the q–function, the estimated wage offer distribution, g_1, *and* the estimated hours distribution, g_2, to sample a choice set $\{(w_s, h_s); s = 1, 2, \ldots, S\}$. Next, one draws for each of the sampled alternatives, (w_s, h_s), a random variable from the Fréchet distribution, say $\epsilon(w_s, h_s)$. Next, it is evaluated which of the drawn alternatives yields highest utility: $\widehat{V}\left(f\left(w_s, h_s; \mathbf{x}_f\right), T - h_s; \mathbf{x}_V\right) \epsilon\left(w_s, h_s\right)$. The alternative (w_r, h_r) thus yielding the highest utility is considered to be the agent's optimal choice according to the model.

4 Data

The model is estimated on the Belgian database of the European Union Statistics on Income and Living Conditions (EU–SILC). We use the data that were collected in 2007. The entire dataset consists of 6348 households or 15493 individuals. It is representative for the Belgian population of private households. Persons living in collective households or institutions are excluded from the target population. The survey provides detailed information on earnings as well as on socio–demographic characteristics of each household.

In order to estimate the model, we relied on three different sub–samples, consisting respectively of couples, single females and single males. Each sub–sample consists exclusively of individuals that are available for the labour market; *i.e.* aged between 16 and 64 year and not being sick, in education, disabled or (pre)retired. Self–employed are excluded due to the lack of reliable information on hours worked and income earned. Mixed households in which only one of the partners is available for the labour market are also excluded from the estima-

Table 1: Descriptive statistics for the estimation sample

	Singles		**Couples**	
Description	*Female*	*Male*	*Female*	*Male*
Age (years)	41.1	39.93	38.08	40.22
% hh having 0-3 year old children	5.78%	0.45%	18.67%	18.67%
% hh having 4-6 year old children	9.46%	0.89%	17.16%	
% hh having 7-9 year old children	10.16%	1.78%	18.19%	
Education:				
Low educated	22.8%	24.5%	16.8%	19.8%
Secondary education	34.6%	41.9%	38.5%	39.0%
High educated	42.6%	33.6%	44.7%	41.2%
Residence:				
Brussels	19.8%	21.2%	9.3%	
Flanders	44.1%	45.2%	58.5%	
Wallonia	36.1%	33.6%	32.3%	
Participation rate (%)	68.12	78.84	79.40	93.20
Hours worked/week:				
Conditional on working	35.88	39.69	32.50	40.84
Unconditional	24.45	31.29	25.81	38.06
Hourly wage (euro)	14.91	15.20	14.73	16.25
Disposable income (€ /month)	1567	1588	3143	
Number of observations	571	449	1457	

Source: Own Calculations, EU–SILC 2007

tion sample. Finally, we drop households whose children are already available for the labour market but are still living with their parents. It is reasonable to assume that their labour supply decisions are different from those of a normal household without working children because it is not clear whether these households consider their labour supply decision as a collective or as an individual process. Given this data selection, we are able to estimate the labour supply model on 1457 couple households, 571 single females, and 449 single males. In total, they represent approximately 41% of the total population in Belgium.

EUROMOD is used as microsimulation tool for the calculation of net disposable income for each element in the opportunity set of households.[17]Gross household labour income is equal to

[17] Version f5.5 was used. More information about EUROMOD can be found at their website:

Table 2: Type specific unemployment rates (%)

	Male			Female		
	Education level			Education level		
Age group	Low	Middle	High	Low	Middle	High
15 to 24 years	26.4	14.0	12.3	33.6	22.1	11.0
25 to 29 years	19.0	7.6	6.9	29.7	13.1	4.8
30 to 34 years	18.0	6.6	3.1	23.5	9.3	3.3
35 to 39 years	11.6	5.3	2.0	21.2	6.9	3.2
40 to 44 years	9.5	4.2	2.9	12.2	6.2	3.0
45 to 49 years	7.4	2.8	2.7	9.3	5.8	2.4
50 to 54 years	7.0	3.7	2.3	10.1	7.0	3.5
55 to 64 years	4.7	3.0	3.0	5.8	7.8	5.3[a]

[a] The exact figure is lacking. The average across all education levels for that age class is taken.
Source: Eurostat Unemployment rates by sex, age and educational attainment level (%), Belgium 2007, downloaded in October 2013.

the sum of labour earnings of all household members. The income tax and employee's social security contributions are deducted from gross income, and social transfers such as social assistance, unemployment benefits, child benefits, education benefits and housing benefits are added. We assume full take–up of social assistance if the eligibility criteria are fulfilled. Descriptive statistics for the selected sub–samples can be found in Table 1. In the wage offer equation an indicator for experience is used. Since we do not have information on the number of years a person has actually been working since she entered the labour market, *potential* experience is used. It is defined as the number of years since the person entered the labour market. That is age *minus* 15 years for a lowly educated person, age *minus* 19 years for a middle educated person, and age *minus* 23 years for a highly educated person. As this variable is highly correlated with age, age will not be included as a separate variable in the wage offer equation.

Besides information from the EU–SILC questionnaire, we also used external information on type specific unemployment. This variable should serve as a proxy for job availability, and may help to identify the contribution of opportunities from preference factors in the model. Table 2 contain the values for these type specific unemployment rates, which are sex and age specific, and dependent on educational attainment level.

https://www.iser.essex.ac.uk/euromod.

5 Estimation results

Table 3 specifies the covariates that have been used in the different parts of the model.

Table 3: Model specification

	Preferences	Opportunities		
	$\mathbf{x}_V$	$\mathbf{x}_{opp}$	$\mathbf{x}_h$	$\mathbf{x}_w$
variable		job offers	hours	wages
Regional dummies[a]	yes	yes	no	no
Education dummies[b]	yes	yes	no	yes
Age	yes	no	no	no
Group specific unemployment rate	no	yes	no	no
Number of children	yes	no	no	no
Gender	yes	yes	yes	yes
Potential experience	no	no	no	yes

[a] Bxl=Brussels, Fl=Flanders, Wal=Wallonia.
[b] Low, Middle, High.

The estimated parameters for the model specification of Table 3 are reported in the *on line* supplementary material Appendix III. Here, we investigate the impact of education on preference intensity for leisure and opportunities.

5.1 Preferences

The marginal rate of substitution between consumption and labour time for singles equals:

$$\mathrm{MRS}_{c,h_j} = \frac{\left(\boldsymbol{\beta}'_{h,j}\mathbf{x}_V\right) \cdot (\ell_j)^{\alpha_{h,j}-1} / T^{\alpha_{h,j}}}{\beta_{c,j} \cdot (c_j)^{\alpha_{c,j}-1}}, \qquad j = 1, 2, \tag{28}$$

and for couples it is equal to:

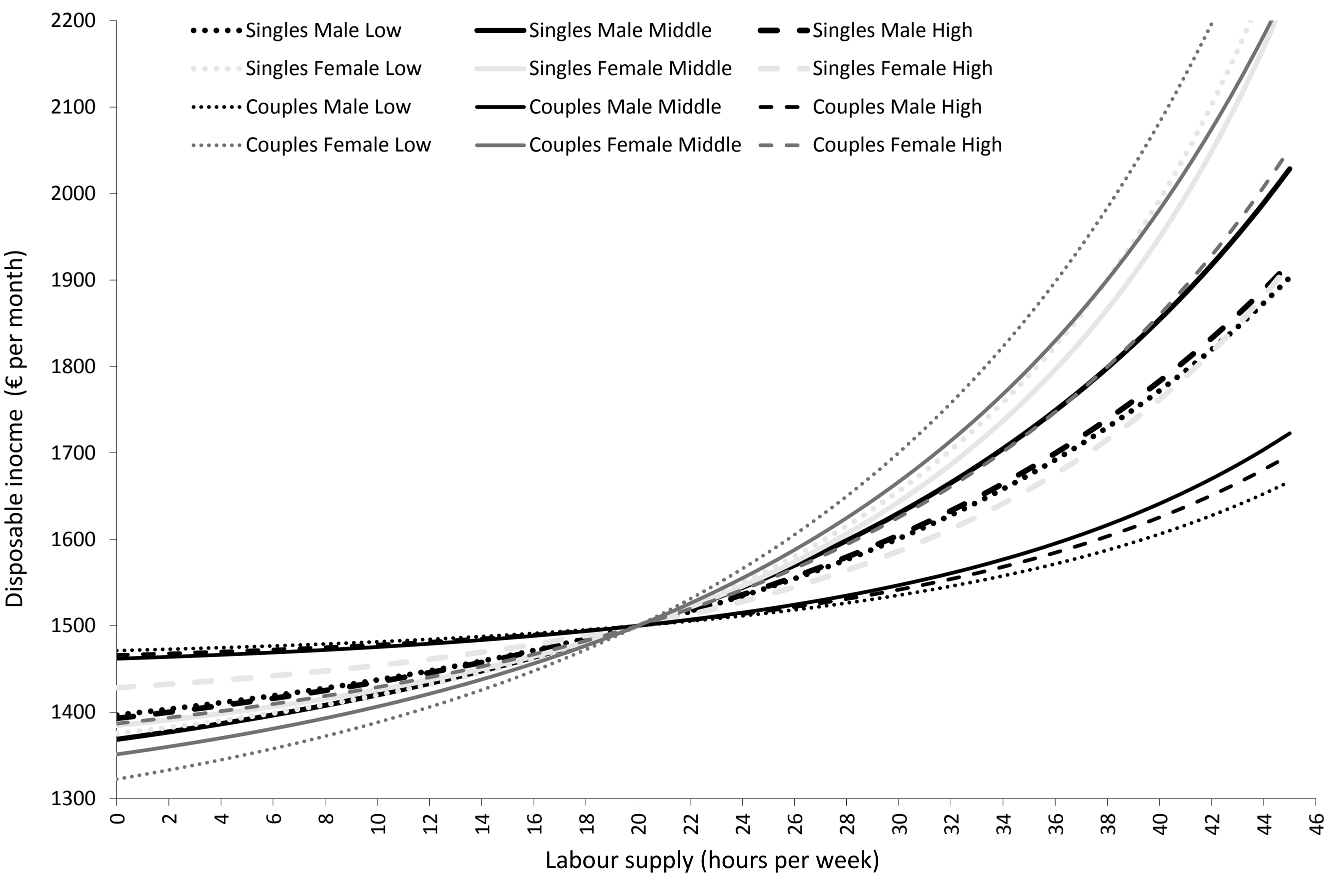
$$\mathrm{MRS}_{c,h_j} = \frac{\left(\boldsymbol{\beta}'_{h_j}\mathbf{x}_V\right) \cdot (\ell_j)^{\alpha_{h_j}-1} / T^{\alpha_{h_j}} + \beta_{h_1,h_2} \left(\frac{(\ell_i/T)^{\alpha_{h_i}}-1}{\alpha_{h_i}}\right) (\ell_j)^{\alpha_{h_j}-1} / T^{\alpha_{h_j}}}{\beta_{c,g} \cdot c^{\alpha_{c,g}-1}}, \tag{29}$$

$$i, j = 1, 2;\ i \neq j.$$

Notice that the covariates influencing preferences affect the marginal rate of substitution only through their influence on $\left(\boldsymbol{\beta}'_{h_j}\mathbf{x}_V\right)$. More specifically, as $\left(\boldsymbol{\beta}'_{h_j}\mathbf{x}_V\right)$ increases in one of the covariates, the marginal rate of substitution in any point (c, h_j) becomes higher for a person with a larger value on that variable. Hence, a person exhibiting a higher value on that covariate will have relatively steeper indifference curves. That is, she will exhibit a more intense preference for leisure relative to consumption.

We illustrate this in Figure 2 for the education level. For males, both in couples (thin black curves in Figure 2), and as singles (fat black curves in Figure 2), the effect of education is non–monotonous. Both highly and lowly educated (dashed, respectively dotted curves) men have less intense preference for leisure relative to consumption, as compared to their fellows with a middle education level. This effect is more outspoken, but much less precisely estimated, in the case of singles. Higher educated females have less intense preferences for leisure relative to consumption, both when living in couples (dark grey curves) or as a single (light grey curves).

Figure 2: Impact of education level on steepness of indifference curves



5.2 Opportunities

Figure 3 represents the estimation of the wage offer distributions. The dotted lines apply to females, the full lines to males. The sex differences are small, as compared to the impact of the other covariates, and not always in the disadvantage of females. The latter is the case for persons with a middle education level.

Higher education shifts the wage offer distribution to the right: compare the fat black, grey, and light grey curves for the distributions at low experience age (10 years), and the thin black, grey, and light grey curves for the impact of education at high experience age (25 years). Potential experience also shifts the wage offer distribution to the right: compare fat and thin lines of the same color. However, when potential experience would exceed the level of 37 years, for males, and 32 years, for females, the wage offer distribution would start shifting to the left again. For experience levels below these bounds, higher education, which implies less potential experience, has therefore two countervailing effects on the wage distribution. The net effect is usually positive, that is a shift of the wage offer distribution to the right. Whether this would also results into an increased acceptance of jobs, depends on income and substitution effects (in preferences), and cannot be fixed *a priori*.

Notice that this is a wage *offer* distribution. Simulated and observed wages of accepted jobs are discussed in Section 6.

Figure 4 represents the distribution of offered labour time regimes.[18] Again, this is not the distribution of are not actual labour time regimes nor that chosen according to the model. The most salient observation is that this distribution is different for males and females, the latter receiving more part time, and less full time job offers.

[18] Admittedly, this part of the model is not non–parametrically identified (see Section 3.2). So, if one feels more for explaining this peak pattern by preferences, we cannot tell him or her empirically wrong.

Figure 3: Estimated wage offer distributions and education

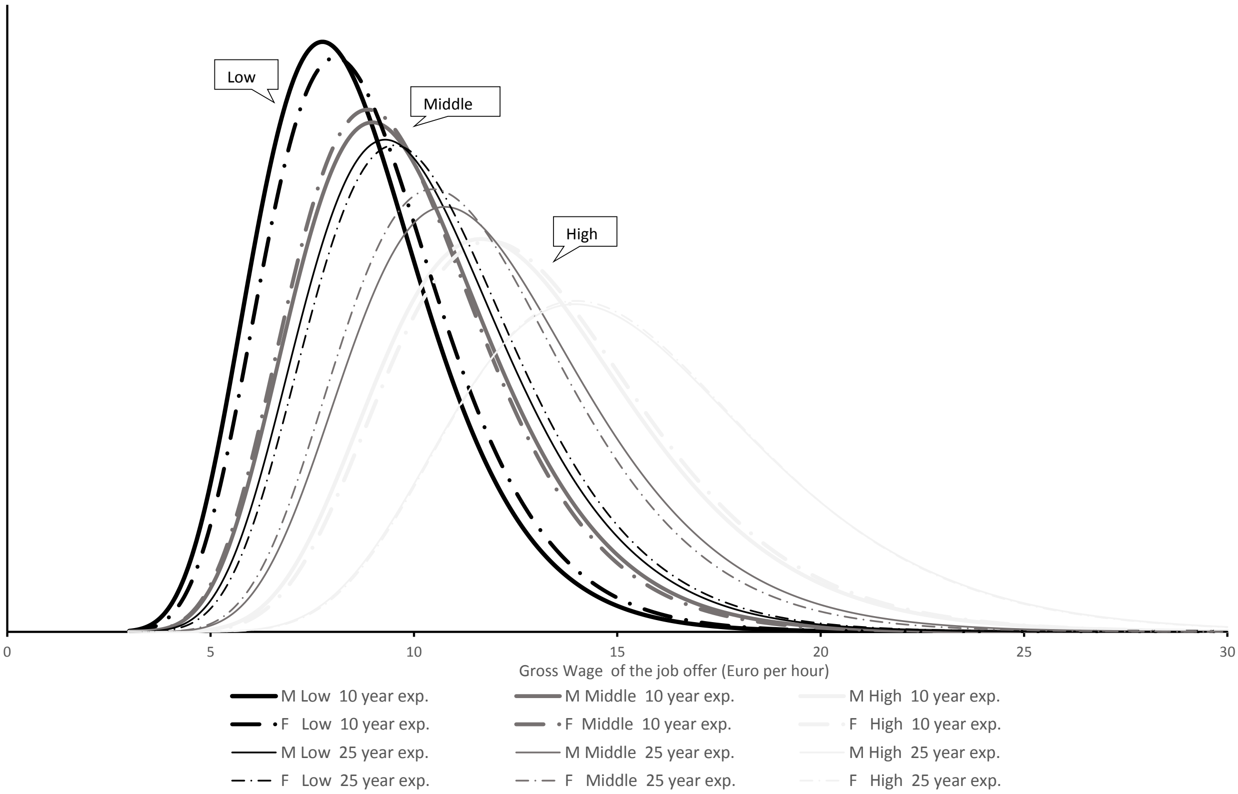


Figure 4: Estimated distribution of offered labour time regimes

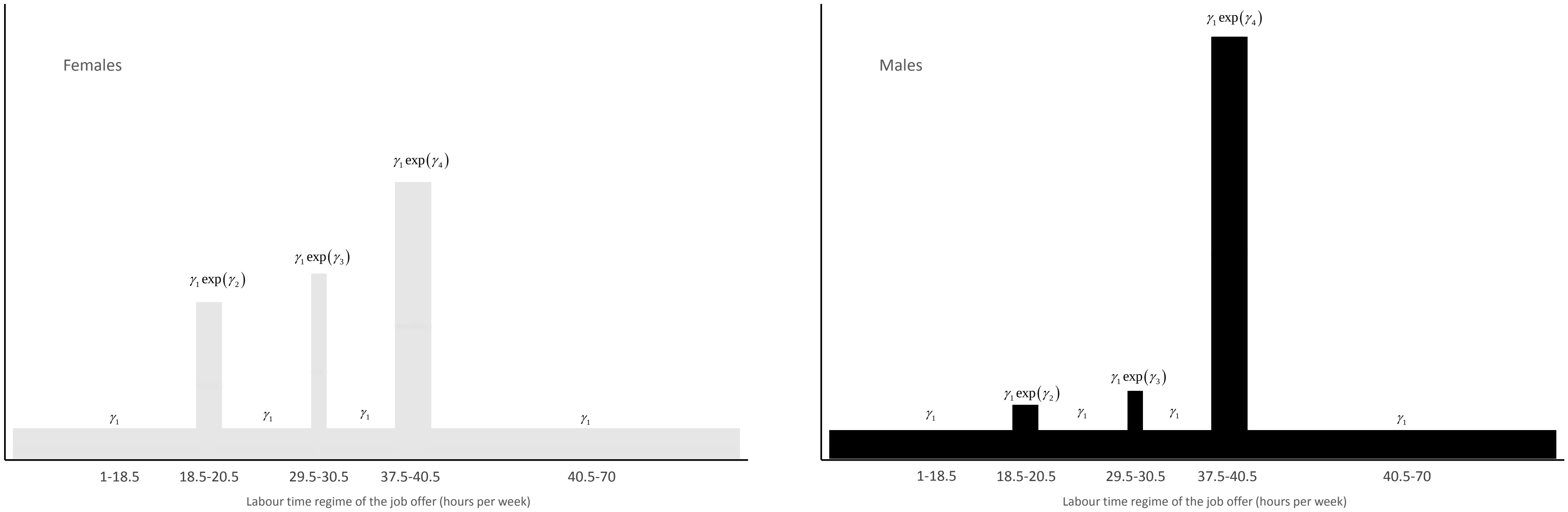
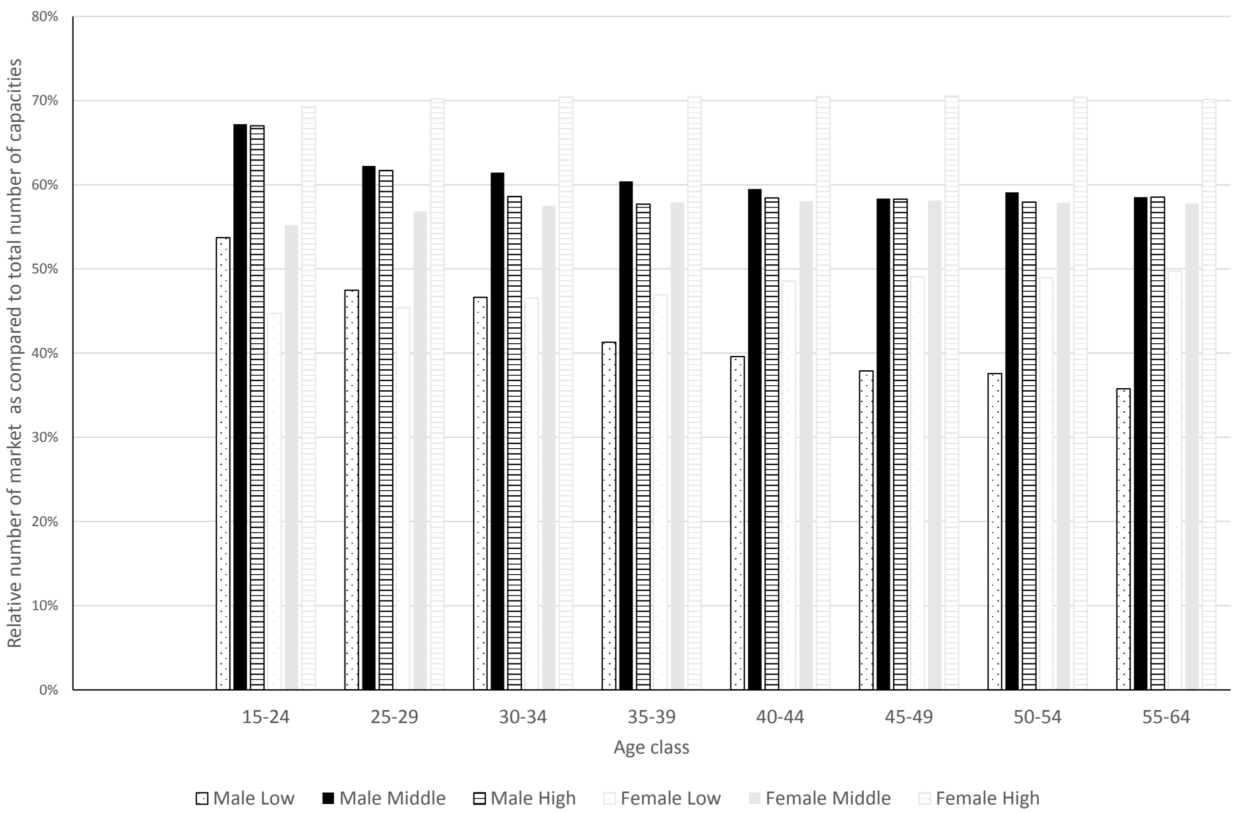


Figure 5 reports the impact of education on the intensity of suitable job offers (the π_1–function). As the type specific unemployment rate varies with education level (and age) we report the joint effect of both. More specifically, for each age class we report the intensity of suitable job offers of males (black bars), respectively females (grey bars) of a lowly (dotted bar), highly educated persons (dashed bar), and those with middle education level (fully coloured bars).
Considered *per se* the education level has a positive effect on the availability of suitable job offers, though the effect of high *versus* middle education is small for males.
From Table 2 it can be seen that the type specific unemployment rate is decreasing in the education level. Now, for males, a higher type specific unemployment rate *in*creases the intensity of suitable job offers. As a consequence, the already small positive effect of education is slightly attenuated, leading ultimately to a net negative effect of high as compared to middle education level for the middle age classes. For females on the other hand the net effect of high education is outspokenly positive, as both the effect of the lower unemployment rate with education and education *per se*, push into the same direction.
Notice that Figure 5 also reveals that the availability of suitable job offers is decreasing slightly with age for males, while we don't see a similar decline for females.

Figure 5: Job offer intensity in function of education and age specific unemployment rate



6 Fit and behavioural response

6.1 Fit

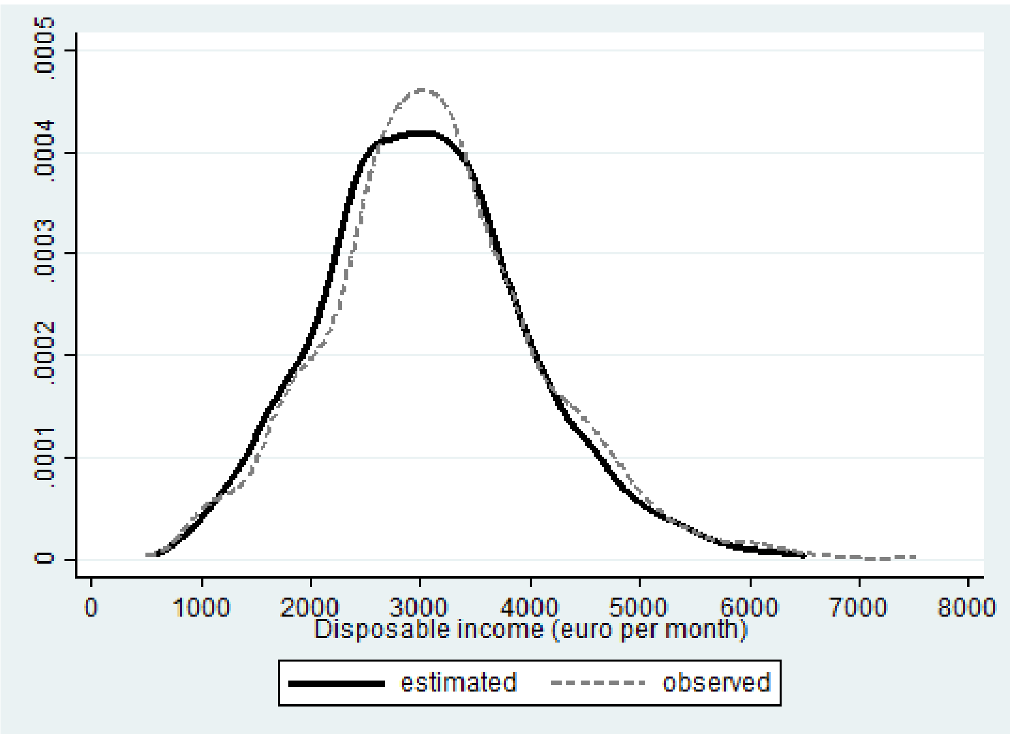
We now evaluate the fit of the estimated model.

1. Couples.

   The mean estimated consumption within couples is 3070 € per month. Compare that with the observed mean of 3143 € per month reported in Table 1. The simulated distribution (black curve in Figure 6) slightly overestimates the number households with lower incomes, at the expense of those with modal disposable incomes (compare the black curve with the observed values represented by the gray dashed one).

   The fit of the (conditional) distribution of female wages is good (RHS panel of Figure 7): the black (simulated) and grey dashed (observed) curve almost coincide. The simulated wage distribution of the males (black line on the LHS panel of Figure 7) is more populated than the observed one (grey dashed line on the LHS panel of Figure 7) at lower and moderately high wages, at the expense of a smaller occurrence of modal and extremely high wages.

Figure 6: Fit disposable income for couples



Curves labelled by `max` refer to the simulated values, while `obs` refer to observed values.

Figure 7: Fit wages males (left) and females (right) in couples

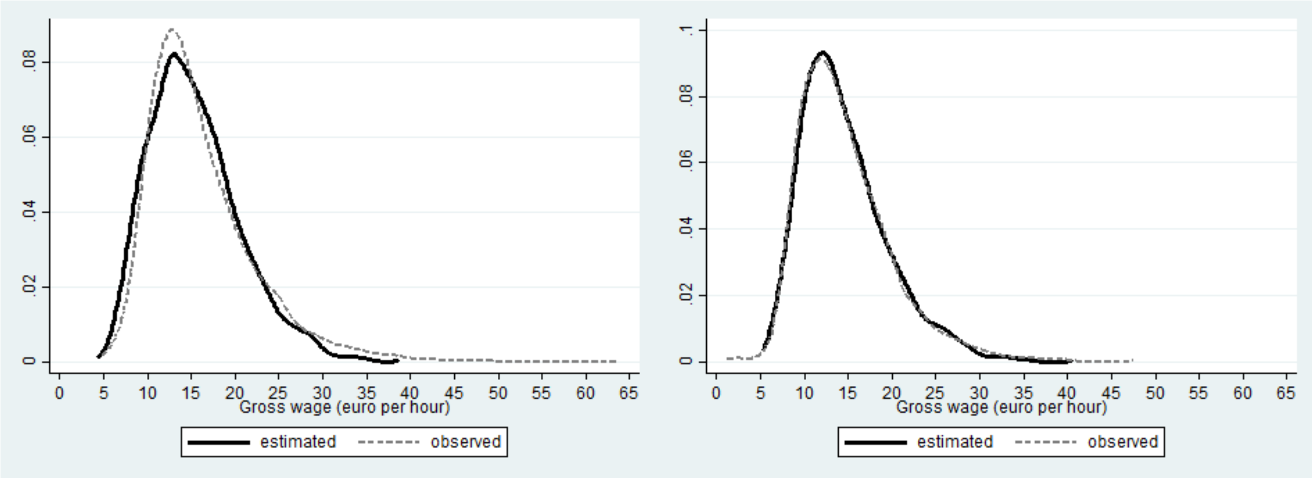
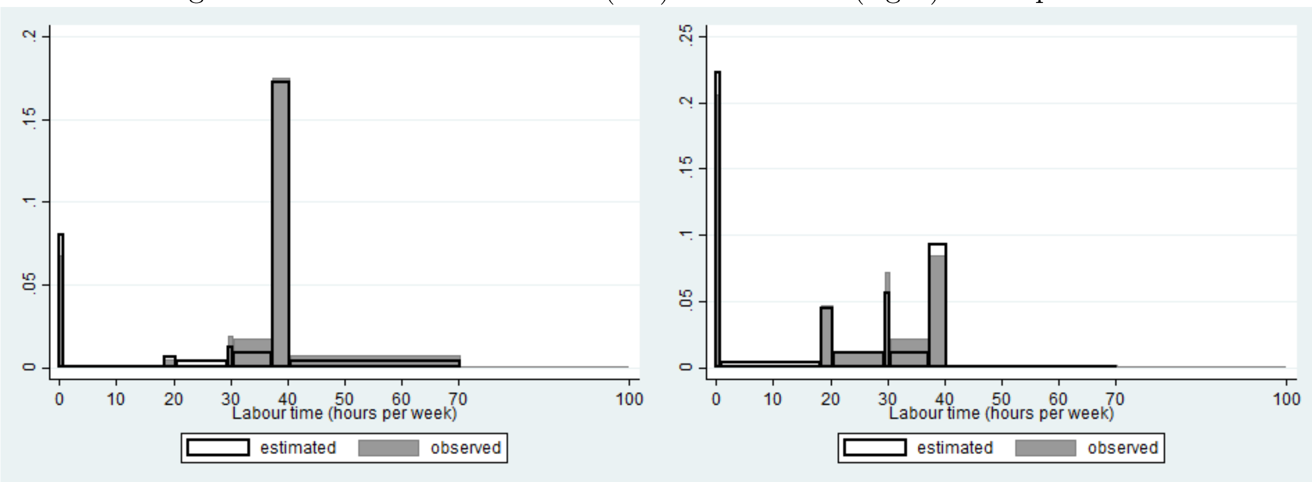


Figure 8: Fit labour time males (left) and females (right) in couples



The number of non–participants in the labour market is overestimated. Compare thereto the unfilled black (observed) and full grey (simulated) left most spike of in both pannels of Figure 8. Still, the number of cases in which none of both partners work, is underestimated. The estimated peaks reasonably well fit the observed values, except for the three quarter time jobs, the occurrence of which is underestimated by the model, both for male and female partners. The percentage of females having a full time job is also slightly overestimated by the model.

2. Singles.

   Figures 9–11 represent the fit of the model for singles. Consumption of single females is reasonably well approximated (RHS of Figure 9). Mean consumption of males is almost perfectly replicated by the estimates (1585 € per month fitted *versus* 1588 € per month observed), but the empirical distribution is somewhat less good approximated, with, amongst other things, an underestimation of the lower tail. The latter is also the case for the single females.

   Similarly, the wage distribution of single females (RHS of Figure 10) is better fitted than that of males (LHS of Figure 10).

   Labour market participation of single males (*cf.* LHS of Figure 11) is overestimated, while that of single females is almost perfectly fitted (RHS of Figure 11). The observed peak for half time jobs is underestimated for males, and that of females for three quarter time jobs is overestimated. The occurrence of full time jobs for males is overestimated. That of females underestimated.

Figure 9: Fit disposable income single males (left) and single females (right)

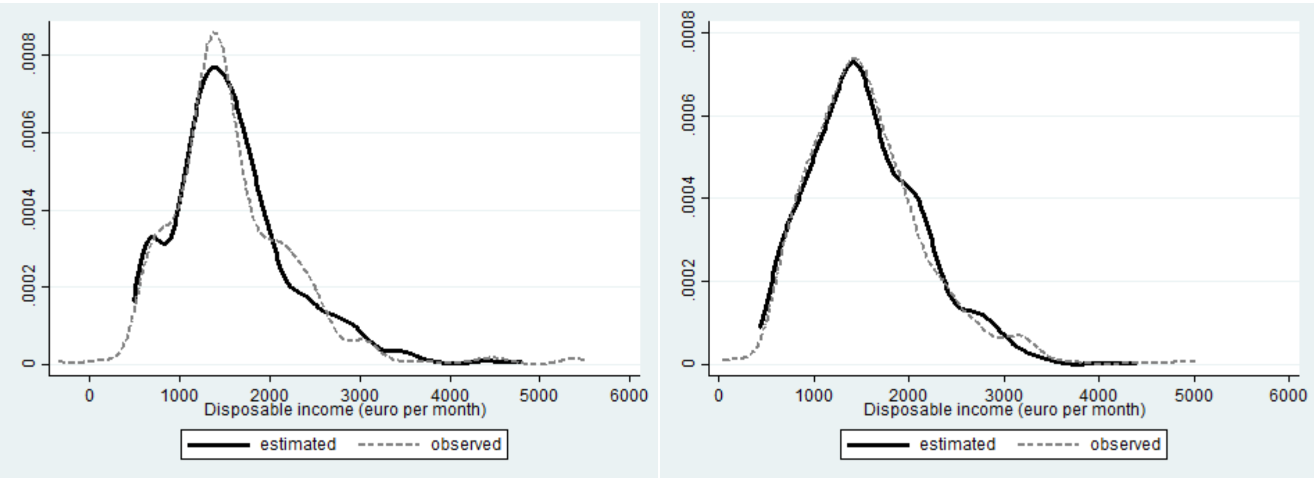


Figure 10: Fit wages single males (left) and females (right)

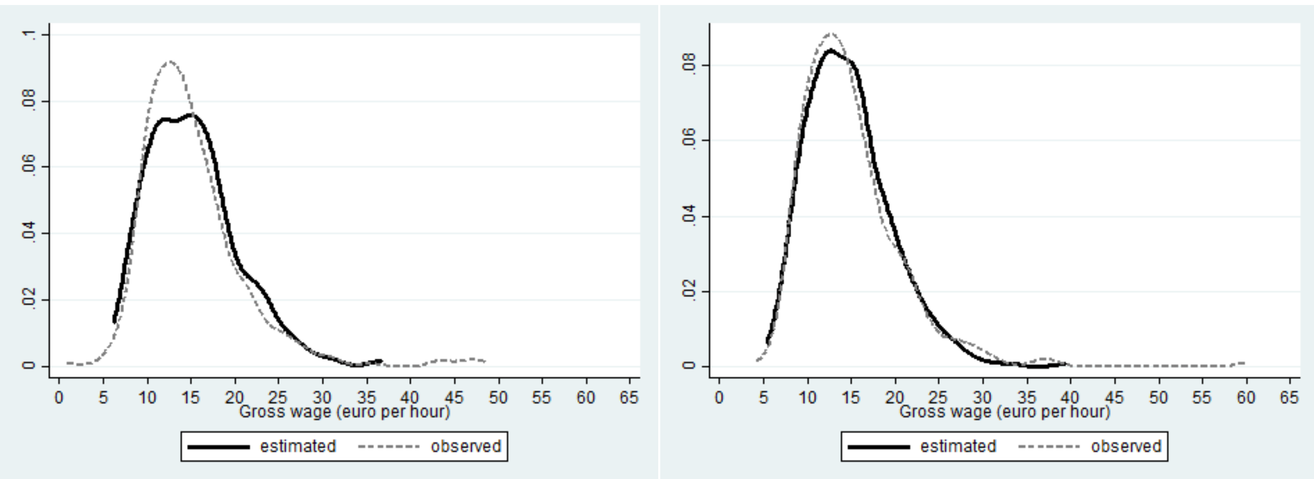
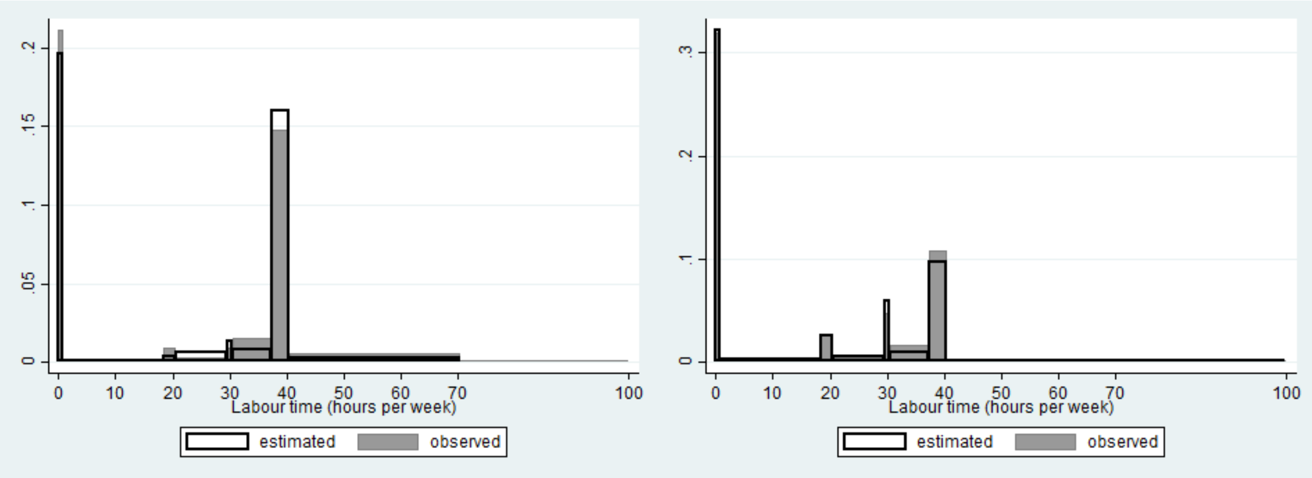


Figure 11: Fit labour time regimes single males (left) and single females (right)



6.2 Elasticities

Table 4 reports the total reaction in terms of labour supply and the effect on participation to the labour market (extensive margin), following a shift of the density of the males', respectively females', wage offer distribution to the right by 10% (augmenting the estimated location parameter with ln 1.1). Additionally, we report intensive margins (effect on labour supply conditional on participating in the base line, inclusive of the labour market leavers). The variable 'part in' gives the percentage of entrants into the labour market, while 'part out' represents the percentage of leavers.

Table 4: Aggregate wage elasticity of labour supply

	Shift of *female* wage distribution			Shift of *male* wage distribution		
	Couple		Single	Couple		Single
	Female	Male	Female	Female	Male	Male
Total elasticity	0.6445	−0.1734	0.6877	−0.2014	0.3304	0.4569
Intensive margin	0.2162	−0.2222	0.1257	−0.2584	0.1365	0.0944
Part in	3.157%	0.480%	3.327%	0.549%	1.716%	2.895%
Part out	0.000%	1.647%	0.000%	1.579%	0.000%	0.000%

Compared to Marshallian elasticities in the literature estimated by static models using micro data, the total elasticity estimates produced here are rather large (Compare *e.g.* with the figures reported in Tables 6 and 7 of Keane, 2011). However, as far as these total elasticities include the extensive margin, and are calculated as the proportional change in total labour time for the whole sub–sample, these need to be compared with macro elasticities, which are usually much larger, even as compared to the figures obtained here.[19] Still, it should be stressed that the figures reported here are conceptually of a different nature, in that actually obtained wages in the present model are the result of choosing the most attractive job offer according to the persons' preferences. Therefore, a reaction to an exogenous change in that wage cannot be conceived of in the framework we used. What was, alternatively, done, is to shift the entire distribution of the wages included in the job offers, to the right. This cannot be considered the same as a change in an exogenously given wage. As the RURO model incorporates frictions due to restrictions in the labour market opportunities an agent faces, this might account for the lower values of the elasticities reported here, compared to values obtained for macro figures.

7 Education and labour market participation

In the present section we present a simulation exercise to assess the full effect of changes in education level on labour market participation and labour supply.

As our sample is not representative for the Belgian population at working age due to selecting only persons available for the labour market, currently not self–employed, we first produced a baseline in which we simulated the education level in accordance with the 2007 distribution, as produced by the Belgian dynamic microsimulation model MIDAS (Federal Planning Bureau).[20] More specifically, we replaced the currently observed education level for each individual in our sample with a randomly assigned education level based on the population figures for the age and gender specific education levels for this baseline (represented in the 'baseline' columns of Table 5).[21] The potential experience and type specific unemployment rate were accordingly adapted. Then, we simulated labour market choices with these new data. Table 6 contains the participation rate (part) and average length of the work week (including non—participants) (h), by gender and age class, resulting from this exercise.

A similar procedure is then followed for the counterfactual scenario on education as obtained from the implementation in the MIDAS–model (see the 'counterfactual' columns of Table 5). Notice that across all age classes, the number of females with higher education in the baseline

[19] On the controversy about micro *versus* macro estimates, see amongst others Chetty *et al.* (2011), Chetty (2012), Fiorito and Zanella (2012), Keane and Rogerson (2012), Jäntti *et al.* (2015), and the references therein.

[20] As MIDAS runs on administrative data that do not include information on education levels, these were imputed, and some basic scenarios were developed to assess their evolution in the future, as new, future generations enter the model. More information on MIDAS can be found in Dekkers *et al.* (2009).

[21] Education levels were assigned on the basis of a comparison of a random draw from the $[0, 1]$–uniform distribution for each case, with the cumulative distribution of education levels in the corresponding gender and age class of the individual.

exceeds that of males by five to ten percentage points. In the counterfactual scenario, the male education distribution is then modelled as approaching that of the females. The distribution of the educational attainment level is thus more or less equal for males and females in the 'counterfactual' columns.

Table 5: Education level distribution by age and sex

Education level	Low		Middle		High	
Scenario	baseline	counterfactual	baseline	counterfactual	baseline	counterfactual
Age			males			
15 – 25	21.20%	20.10%	40.15%	39.90%	38.65%	39.99%
26 – 30	27.42%	19.84%	41.27%	40.00%	31.31%	40.16%
31 – 35	27.27%	19.44%	40.41%	40.13%	32.32%	40.43%
36 – 40	27.66%	20.47%	40.22%	39.75%	32.11%	39.78%
41 – 45	26.87%	19.80%	39.22%	40.24%	33.92%	39.96%
46 – 50	26.62%	20.33%	39.68%	39.97%	33.71%	39.70%
51 – 55	26.12%	20.50%	38.15%	39.84%	35.73%	39.67%
56 – 60	25.35%	20.15%	38.79%	39.95%	35.86%	39.90%
61 – 65	24.15%	19.96%	38.66%	39.70%	37.18%	40.33%
all	27.27%	20.90%	41.54%	41.42%	31.19%	37.68%
			females			
15 – 25	20.36%	20.21%	40.23%	40.12%	39.41%	39.68%
26 – 30	20.22%	19.51%	39.77%	40.38%	40.01%	40.11%
31 – 35	18.71%	20.26%	38.09%	40.12%	43.20%	39.62%
36 – 40	19.77%	19.31%	38.35%	39.69%	41.89%	41.00%
41 – 45	19.61%	20.14%	38.03%	39.68%	42.36%	40.19%
46 – 50	19.35%	20.46%	38.27%	39.58%	42.38%	39.96%
51 – 55	19.14%	20.12%	38.03%	38.80%	42.83%	41.08%
56 – 60	20.03%	19.33%	38.73%	40.48%	41.24%	40.19%
61 – 65	19.46%	20.11%	39.01%	39.80%	41.53%	40.09%
all	20.52%	20.79%	40.26%	41.17%	39.22%	38.04%

Source: MIDAS implemented scenarios for baseline coinciding with 2007 and counterfactual aiming at a catch–up of female's higher education level than males' in the baseline, by 2050.

Accordingly, we can learn from Table 6 that the overall effect of changing education levels over time is mainly concentrated among the men. More specifically, labour market participation of *single* males between 30 and 45 years old increases by 1.5 to over 7 percentage points. The mean number of hours worked per week for the same categories increases with half an hour to more than four hours. Female labour market participation and labour supply are not fundamentally affected in the alternative situation, as female education in the counterfactual does not differ much from that in the baseline. I anything, female labour market participation and labour time decrease slightly in the alternative scenario.

All in all, the model predicts that expected shifts in the education level will not contribute much to the change in labour market participation.

Table 6: Participation and mean labour time by age class in baseline and counterfactual

Age	$15-25$	$26-30$	$31-35$	$36-40$	$41-45$	$46-50$	$51-55$	$56-60$	$61-65$	all
					Couples: males					
n obs	68	182	247	290	243	132	148	90	16	1457
part baseline	89.7%	90.1%	88.7%	94.1%	92.2%	94.8%	93.2%	74.4%	87.5%	90.9%
part counterf.	89.7%	92.3%	89.1%	94.5%	93.4%	96.0%	93.2%	80.0%	87.5%	92.0%
h baseline	33.4	34.3	35.9	38.8	37.2	38.3	37.4	28.2	35.8	36.4
h counterf.	33.4	35.1	36.2	39.0	37.6	38.6	37.2	30.2	35.8	36.7
					Couples: females					
n obs	124	241	264	260	230	172	99	58	9	1457
part baseline	74.2%	78.8%	73.5%	78.8%	82.2%	80.8%	73.7%	72.4%	55.6%	77.5%
part counterf.	73.4%	77.2%	73.5%	78.1%	81.3%	80.2%	75.8%	69.0%	55.6%	76.8%
h baseline	24.3	25.4	23.2	24.7	26.0	24.9	20.9	22.7	18.1	24.4
h counterf.	24.2	24.7	23.0	24.4	25.6	24.3	21.6	22.1	18.1	24.0
					Singles: females					
n obs	43	59	83	97	89	82	55	51	12	571
part baseline	58.1%	69.5%	69.9%	66.0%	65.2%	79.3%	80.0%	70.6%	50.0%	69.5%
part counterf.	58.1%	69.5%	68.7%	66.0%	65.2%	78.0%	80.0%	70.6%	50.0%	69.2%
h baseline	18.4	23.6	27.1	23.6	21.9	27.1	24.8	21.8	14.3	23.7
h counterf.	18.4	23.6	26.6	23.6	21.9	26.8	24.8	21.8	14.3	23.6
					Singles: males					
n obs	46	60	67	68	54	65	46	33	10	449
part baseline	80.4%	88.3%	91.0%	77.9%	85.2%	87.7%	67.4%	72.7%	50.0%	80.6%
part counterf.	80.4%	83.3%	92.5%	85.3%	87.0%	87.7%	69.6%	72.7%	60.0%	83.1%
h baseline	31.5	30.3	34.6	30.0	32.8	33.2	23.3	25.2	18.5	30.4
h counterf.	31.5	32.1	35.1	34.2	33.6	33.3	24.7	25.2	22.1	31.7

In Table 7 we give an idea of the relative contribution preferences and opportunities to the explanation of this moderate shift. Recall from Figure 2 that males with lower and higher education level have *more* intense preferences for leisure, than those with middle education levels, while the intensity of preferences for leisure of females is monotonically decreasing in the education level. On the other hand, the intensity of suitable job offers is increasing with education level, but there is a countervailing effect of higher education on the wage

Table 7: Impact of education through preferences and opportunities

	Couples		Singles	
	Males	Females	Males	Females
n obs	1457	1457	449	571
part base	90.87%	77.48%	80.62%	69.53%
part alt pref	90.94%	77.62%	81.51%	69.53%
part alt opp	91.90%	76.94%	82.85%	69.35%
part counterf.	91.97%	76.80%	83.07%	69.17%
h base	36.35	24.35	30.37	23.72
h alt pref	36.27	24.32	30.82	23.72
h alt opp	36.78	24.12	31.38	23.67
h counterf.	36.73	24.04	31.67	23.60

distribution as potential experience will decrease. In Table 7 the figures for participation (part) and labour time (h) labelled by 'base' and 'counterf.' repeat the population labour participation and average labour time (hours per week) for the corresponding scenarios from the last column of Table 6. The rows labelled by 'alt pref' use the education level in the preferences according to the alternative scenario (a shift from low to high for men, a small increase of middle at the expense of high for females, see Table 5), but keep the wage offer distribution and job offer intensity corresponding to the education levels in the baseline. Except for single males, participation figures are hardly affected by this change in education level through preferences. Participation of single males increases with almost one percentage point and they work on average half an hour longer. This cannot easily be explained by the indifference curves (as the high and low educated males have almost identical indifference curves), but must be the consequence of the interaction of wage and income effects which might be different at different utility levels.

The rows with label 'alt opp' correspond to figures obtained by the wage offer distribution and job offer intensity with education levels as in the alternative scenario, while keeping the education profile of the baseline along the preference side of the model. Participation and labour time of females decreases slightly, what means that the increased potential experience gain due to lower education does not counterbalance the effect of lower intensity of job offers. Participation of men in couples increases with one percentage point and that of single males with more than two percentage points. Average labour time of the latter is almost one hour higher for single males, but increases only half an hour for males in couples. Also for males, the lower potential experience age due to higher education does not seem to counterbalance the effect of higher job offer intensity.

We conclude that the already small change in labour market participation due to expected changes in education level, run almost completely through the channel of opportunities rather than through preferences.

8 Conclusion

In the present paper we have explained how to estimate a RURO model. Next, we illustrated how the model can be used in simulation exercises.

We think the RURO framework might prove useful in modelling behavioural reactions to tax–benefit reforms as assessed by micro–simulation models. Moreover it provides a tool to throw some light on the extent to which the impact of such reforms runs through preferences, or rather through the channel of modifying opportunities. The latter is not only a question of modifications in the budget set, but also the availability of jobs in accordance with a person's capacities comes into play. As this distinction is at the heart of some policy debates, such as the extent to which a tax shift from labour to consumption might create more jobs, we feel the approach presented here is also relevant from a policy point of view.

Of course there are some limitations too. The model is essentially static. And it does not provide a complete equilibrium framework. It is not a matching model in which job offers are matched (or not) to suitable candidates. Frictions on the labour market are taken as given. Within this containment however, it is possible to make a distinction between the impact of measures that runs through preferences, and the part that can be attributed to opportunities.

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Appendices to "Estimating and simulating with a random utility–random opportunity model of job choice"

By Bart Capéau, André Decoster, and Gijs Dekkers

Appendix I Poisson processes

Originally, a Poisson process is a stochastic process describing the probability of the number of occurrences of a particular event during a certain time spell. More specifically, a Poisson process assumes that distribution of the time between each pair of consecutive events is independent from the moment at which the first of these two events occurred, or from any other event in the past, and that these inter–arrival times are exponentially distribution with parameter λ. This parameter λ measures the *intensity* with which such events occur. Under these assumptions, the probability that a certain event occurs n times within a given unit of time, where $n \in \{0, 1, 2 \ldots\}$, equals:

$$P\left(N\left(t+1\right)-N\left(t\right)=n\right)=\frac{\lambda^n \exp\left[-\lambda\right]}{n!}, \tag{A.1}$$

where $N(t)$ is the number of events that occurred in total after t units of time. A Poisson process is *inhomogeneous* if the intensity parameter depends on the moment of measurement, $\lambda(t)$ say. In that case, the probability that n events occur within a time interval $[t, t+\tau]$, equals:

$$P\left(N\left(t+\tau\right)-N\left(t\right)=n\right)=\frac{\left(\Lambda\left(\tau\right)\right)^n \exp\left[-\Lambda\left(\tau\right)\right]}{n!}, \tag{A.2}$$

where $\Lambda(\tau) := \int_t^{t+\tau} \lambda(s) \, \mathrm{d}\, s$, which is, because of the time independence property of Poisson processes, independent of t.

A Poisson process can also be *spatial*. Let an event be described as a point in an m–dimensional space. A spatial Poisson process determines the probability that n events occur within a subset of the m–dimensional space. Let, for example, $\mathcal{B} \subset \mathbb{R}^m$, and let $N(\mathcal{B})$ be the number of events occurring in $\mathcal{B}$. If the occurrence of such events obeys a Poisson process, then the probability that there occur n events in $\mathcal{B}$, equals:

$$P\left(N\left(\mathcal{B}\right)=n\right)=\frac{\lambda^n \exp\left[-\lambda\right]}{n!}. \tag{A.3}$$

Again, such a process is said to be inhomogeneous if the intensity of occurrence depends on the points $x \in \mathbb{R}^m$. To describe that process, assume that there exists a measure ρ defined on (measurable) subsets of the space $\mathbb{R}^m$, and that $\rho(\mathcal{B}) = 1$. The probability that there occur n events in the subset $\mathcal{B}$, is then:

$$P\left(N\left(\mathcal{B}\right)=n\right)=\frac{\left(\Lambda\left(\mathcal{B}\right)\right)^n \exp\left[-\Lambda\left(\mathcal{B}\right)\right]}{n!},. \tag{A.4}$$

where $\Lambda(\mathcal{B}) := \int_{x\in\mathcal{B}} \lambda(x) \, \mathrm{d}\, \rho(x)$.

Job offers and the availability of non–market activities each are described by an inhomogeneous spatial Poison process in RURO models. These processes are independent. However, given that the RURO model is static, the stock of capacities an individual is endowed with, is assumed to be fixed. If demand for these capacities (by means of job offers) intensifies, a relatively smaller amount of these capacities serves exclusively for executing non–market activities.

Appendix II Likelihood: the case of couples

Assumed that both partners have identical tastes specified over household consumption, each of the partners' leisure time, and other attributes associated with the activities the partners executed. From these, a utility function, say $\mathbf{\Psi}$ can be derived in terms of both spouses' wages and hours of labour time, (w_1, h_1, w_2, h_2). If one assumes that each partner's process of job offer arrivals and availability of non–market alternatives is independent of that of the other, the following expressions for the likelihood are obtained, for the cases respectively that both partners, only one of them, or none of both will accept a job offer:

$$
\begin{aligned}
\varphi^c(w_1, h_1, w_2, h_2) &= \frac{\mathbf{\Psi}(w_1,h_1,w_2,h_2) \prod_{j=1,2} q_j\, g_1^j(w_j) g_2^j(h_j)}{\mathbf{\Psi}(0,0,0,0)\ +A+B+C}, \\
\varphi^c(w_1, h_1, 0, 0) &= \frac{\mathbf{\Psi}(w_1,h_1,0,0)\, q^1\, g_1^1(w_1) g_2^1(h_1)}{\mathbf{\Psi}(0,0,0,0)\ +A+B+C}, \\
\varphi^c(0, 0, w_2, h_2) &= \frac{\mathbf{\Psi}(0,0,w_2,h_2)\, q^2\, g_1^2(w_2) g_2^2(h_2)}{\mathbf{\Psi}(0,0,0,0)\ +A+B+C}, \\
\varphi^c(0, 0, 0, 0) &= \frac{\mathbf{\Psi}(0,0,0,0)}{\mathbf{\Psi}(0,0,0,0)\ +A+B+C},
\end{aligned}
\tag{16''}
$$

with:

$$A := \int_{r_1 \in \mathcal{W}} \int_{t_1 \in \mathcal{H}} \mathbf{\Psi}(r_1, t_1, 0, 0)\ q_1\, g_1^1(r_1)\, g_2^1(t_1)\ \mathrm{d}\,t_1\ \mathrm{d}\,r_1,$$

$$B := \int_{r_2 \in \mathcal{W}} \int_{t_2 \in \mathcal{H}} \mathbf{\Psi}(0, 0, r_2, t_2)\ q_2\, g_1^2(r_2)\, g_2^2(t_2)\ \mathrm{d}\,t_2\ \mathrm{d}\,r_2,$$

$$C := \int_{r_1 \in \mathcal{W}} \int_{t_1 \in \mathcal{H}} \int_{r_2 \in \mathcal{W}} \int_{t_2 \in \mathcal{H}} \mathbf{\Psi}(r_1, t_1, r_2, t_2) \prod_{j=1,2} q_j\, g_1^j(r_j)\, g_2^j(t_j)\ \mathrm{d}t_2\, \mathrm{d}r_2\, \mathrm{d}t_1\ \mathrm{d}\,r_1.$$

where q^j, g_i^j $(i = 1, 2)$ are the relative intensity of job offers, the wage offer density, and the labour time density of partner j $(j = 1, 2)$. For couples, a non–market alternative is an alternative in which none of both partners is engaged in the formal labour market.

Appendix III Coefficient estimates

Table A.I: Preferences couples

Log likelihood		-8482.1758	
Description	Estimate	Standard Error	t–value
1.a) Consumption & leisure interaction M&F			
Consumption Couples exponent	0.610	0.051	11.96
Consumption Couples constant	4.873	0.310	15.70
Leisure interaction M&F.in couples	0.206	0.077	2.69
Consumption single M exponent	0.292	0.123	2.38
Consumption single M constant	4.740	0.395	12.00
Consumption single F exponent	0.049	0.149	0.33
Consumption single F constant	4.181	0.338	12.36
1.b) Leisure coefficients males in couples			
Leisure M in couples exponent	-8.351	0.663	-12.59
Leisure M in couples constant	20.959	7.880	2.66
Leisure M in couples ln(age)	-11.339	4.321	-2.62
Leisure M in couples $\ln(\text{age})^2$	1.591	0.601	2.65
Leisure M in couples ch03	0.007	0.059	0.12
Leisure M in couples ch36	0.078	0.063	1.23
Leisure M in couples ch69	-0.009	0.058	-0.15
Leisure M in couples dum region Walloon[a]	0.132	0.068	1.94
Leisure M in couples dum region Brussels	0.168	0.112	1.49
Leisure M in couples dum education LOW[b]	-0.174	0.085	-2.05
Leisure M in couples dum education HIGH	-0.078	0.060	-1.31
1.c) Leisure coefficients females in couples			
Leisure F in couples exponent	-6.995	0.502	-13.93
Leisure F in couples constant	32.068	14.700	2.18
Leisure F in couples ln(age)	-18.521	8.368	-2.21
Leisure F in couples $\ln(\text{age})^2$	2.879	1.197	2.40
Leisure F in couples ch03	0.550	0.179	3.08
Leisure F in couples ch36	0.533	0.187	2.84
Leisure F in couples ch69	0.426	0.191	2.23
Leisure F in couples dum region Walloon	0.302	0.173	1.74
Leisure F in couples dum region Brussels	0.062	0.247	0.25
Leisure F in couples dum education LOW	0.612	0.331	1.85
Leisure F in couples dum education HIGH	-0.753	0.183	-4.10

[a] Flanders region is reference category; [b] Middle education level is reference category.

Table A.I: Preferences singles

Description	Estimate	Standard Error	t–value
1.d) Leisure coefficients single males			
Leisure single M exponent	-5.444	1.002	-5.43
Leisure single M constant	36.394	23.737	1.53
Leisure single M ln(age)	-20.375	13.239	-1.54
Leisure single M ln(age)^2	3.024	1.865	1.62
Leisure single M ch36	-0.457	1.112	-0.41
Leisure single M ch69	-1.135	0.698	-1.63
Leisure single M dum region Walloon	0.951	0.425	2.24
Leisure single M dum region Brussels	0.262	0.372	0.70
Leisure single M dum education LOW	-0.581	0.387	-1.50
Leisure single M dum education HIGH	-0.502	0.335	-1.50
1.e) Leisure coefficients single females			
Leisure single F exponent	-7.688	0.977	-7.87
Leisure single F constant	62.678	23.311	2.69
Leisure single F ln(age)	-34.609	12.929	-2.68
Leisure single F ln(age)^2	4.876	1.809	2.70
Leisure single F ch03	0.838	0.502	1.67
Leisure single F ch36	0.128	0.239	0.54
Leisure single F ch69	-0.141	0.196	-0.72
Leisure single F dum region Walloon	0.212	0.199	1.06
Leisure single F dum region Brussels	-0.258	0.188	-1.37
Leisure single F dum education LOW	0.133	0.326	0.41
Leisure single F dum education HIGH	-0.616	0.217	-2.84

Table A.I: Opportunities, Relative intensity of market alternatives, peaks hours, and wage offer distribution

Description	Estimate	Standard Error	t–value
2.a) Estimated coefficients opportunities & peaks males			
Opportunity M constant	-4.488	0.247	-18.19
Opportunity M unemployment rate	0.338	0.226	1.50
Opportunity M dummy region Walloon	-0.547	0.223	-2.45
Opportunity M dummy region Brussels	-1.215	0.285	-4.27
Opportunity M dummy LOW education	-0.987	0.277	-3.56
Opportunity M dummy HIGH education	0.049	0.265	0.19
Peaks M <18.5,20.5> interval	0.643	0.229	2.81
Peaks M <29.5,30.5> interval	0.862	0.189	4.55
Peaks M <37.5,40.5> interval	2.690	0.060	45.17
2.b) Estimated coefficients opportunities & peaks females			
Opportunity F constant	-4.300	0.185	-23.19
Opportunity F unemployment rate	-0.072	0.124	-0.58
Opportunity F dummy region Walloon	-0.394	0.157	-2.51
Opportunity F dummy region Brussels	-0.783	0.219	-3.58
Opportunity F dummy LOW education	-0.339	0.217	-1.56
Opportunity F dummy HIGH education	0.522	0.195	2.68
Peaks F <18.5,20.5> interval	1.636	0.100	16.42
Peaks F <29.5,30.5> interval	1.804	0.108	16.69
Peaks F <37.5,40.5> interval	2.206	0.070	31.36
3. Estimated coefficients wage equations			
3.a) Wage equation males			
Wage M σ	0.264	0.004	60.63
Wage M constant	2.066	0.029	72.00
Wage M potential experience	2.297	0.244	9.41
Wage M potential experience^2	-3.110	0.545	-5.71
Wage M low education	-0.147	0.019	-7.79
Wage M high education	0.260	0.015	17.39
3.b) Wage equation females			
Wage F σ	0.261	0.004	59.05
Wage F constant	2.043	0.026	77.61
Wage F potential experience	2.457	0.239	10.30
Wage F potential experience^2	-3.869	0.592	-6.54
Wage F low education	-0.095	0.023	-4.08
Wage F high education	0.291	0.016	18.61